Precalculus Review
Handout 1.1
Real Numbers and the Real Number Line

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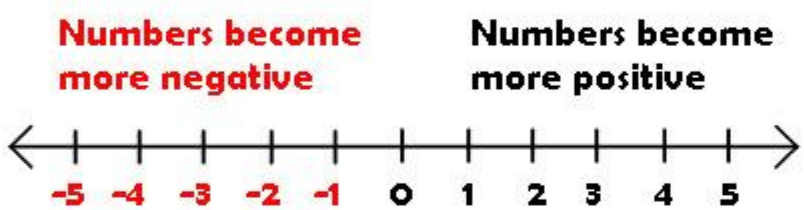
Real numbers are represented on the real number line, or x-axis, on the coordinate system. A real number corresponds to a point on the real number line and is the coordinate of that point. When drawing the real number line, points whose coordinates are integers (whole numbers such as 3, 15, -24, etc.) are usually written out.

The point "o" (zero) on the real number line is the **origin**.



The numbers **increase** in value as the direction moves to the **right** of the origin and are **positive**.

The numbers **decrease** in value as the direction moves to the **left** of the origin and are **negative**.

Nonnegative refers to numbers that are **positive or zero**.

Nonpositive refers to numbers that are **negative or zero**.

A **rational number** is a real number that can be written as a ratio of two integers (such as ½ = 0.5 or $\frac{7}{5}$ = 1.4). The rational numbers can be either **repeating decimals** (e.g., $\frac{4}{11} = 0.\overline{36}$) or **terminating decimals** (e.g., $\frac{5}{8}$ = 0.625)

Irrational numbers are real numbers represented by decimal approximations since they **cannot** be represented as repeating or terminating decimals.

We are already familiar with a few:

π = 3.14159265359
e = 2.71828182846
$\sqrt{5}$ = 2.23606798

Problem 1.1A and 1.1B

Given the following repeating decimals, write them as a ratio of two integers.

A. $0.\overline{63}$
B. $0.3\overline{18}$

So this is similar to our example above on repeating decimals. So we know that **a**. repeats to 0.6363…

Now we want to convert it to a ratio, it's fraction form.

Step 1: We need to set the decimal point to the left and right of the repeating decimal. What this means is that we put the decimal point to the left of the repeating numbers and then put it to the right of the repeating numbers. Take a look at the pattern to see what repeats.

We see that the pattern is "63" repeating. We focus on putting the decimal point to the left of "6" and to the right of "3."

Step 2: Place the decimal points to the left and to the right of the repeating numbers.

Let **x** equal the decimal placed to the left of the repeating numbers. The first example is easy since it's already placed in position.

x = 0.6363

For the right, we already know that the “63” repeats. We want to place the decimal to the right of the repeating pattern. What can we do? We multiply both sides by 100 to turn 0.6363… into 63.6363… and x will now become 100x.

100x = 63.6363

Step 3: Subtract the first equation from the second equation.

$$\begin{array}{r} 100x=63.6363 \\ -\quad x=0.6363 \\ \hline 99x=63 \end{array}$$

Subtract the equation.

$$\frac{\cancel{99}x}{\cancel{99}x}=\frac{63}{99}$$

Divide both sides by 99.

$$x = \frac{63}{99}=\frac{7}{11}$$

Simplify by dividing the top and bottom by 9 since they’re both divisible by 9.

If you have a calculator you can check your answer and you’ll see that $\frac{7}{11}$ equals 0.636363…

Problem 1.1B

Now we move on to B. Now we’ve reached an obstacle. We can see that the 1818… repeats, but now there’s a “3” in front of it! It’s like watching a movie and someone with a big hat sits right in front of you.

Don’t worry, the process is still the same from Problem 1.1A.

Step 1: Identify the repeating numbers.

We see that “18” are the repeating numbers. However, there’s a “3” in front of them so we cannot just let x equal the original number. We will now have to find a way to set the decimal point to the left of the “18.”

Step 2: Place the decimal points to the left and to the right of the repeating numbers.

If x = $0.3\overline{18}$, how can we place the decimal point to the left? Simple, we multiply both sides by 10. We get:

10x = 3.1818

For the right, we have to place the decimal point after the "8." We would have to move the decimal point three places to the right. To do that, we multiply both sides by 1000. We get:

1000x = 318.1818

Step 3: Subtract the first equation from the second equation.

$$\begin{array}{r} 1000x = 318.1818 \\ -\quad 10x = 3.1818 \\ \hline 990x = 315 \end{array}$$

Subtract the equation.

$$\frac{\cancel{990x}}{\cancel{990x}} = \frac{315}{990}$$

Divide both sides by 990.

$$x = \frac{315}{990} = \frac{7}{22}$$

Simplify by dividing the top and bottom by the least common multiple. Since many won't have access to a calculator during an exam, use small numbers (such as 2 through 9) until it becomes smaller and can't find a common multiple anymore.

Let's give it a try. We can see that we can divide both by 3.

$$x = \frac{315}{990} = \frac{105}{330}$$

We see that the numbers are still large so now we can simplify some more. We see that we can still use "3", but now we can also use "5" a larger number. After doing that, we can see that they are both divisible by "3." We divide again and get our final answer.

$$= \frac{105}{330} = \frac{21}{66} = \frac{7}{22}$$

If you have a calculator you can check your answer by dividing the final answer $\frac{7}{22}$ and we get 0.318181818....