Precalculus/Algebra Review
Handout A.1
Factoring Cubic Polynomials

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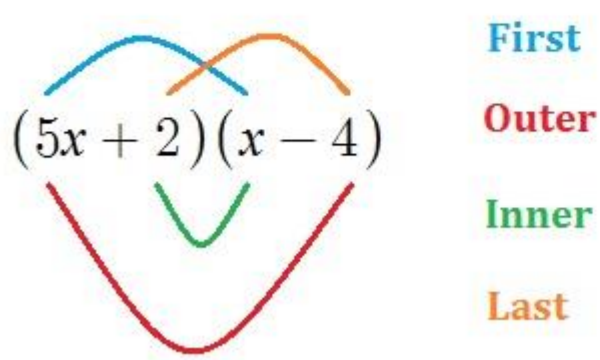
Quick Review: FOIL Method

In algebra, many people are familiar with multiplying binomials using the FOIL method. I am going to cover a few points regarding this topic since this material comes back in future mathematics courses such as calculus.

Example 1

Let's look at an example:

$$(\mathbf{5x+2})(\mathbf{x-4})$$



Under the FOIL method, the order in which you multiply follows the diagram above: First, Outer, Inner, and Last.

First: $(5x)(x) = 5x^2$

Outer: $(5x)(-4) = -20x$

Inner: $(2)(x) = 2x$

Last: $(2)(-4) = -8$

If the outer and inner terms have the same variable, you perform the addition or the subtraction of the terms. In our case, the outer and inner do have the same terms. The outer is negative and the inner is positive: $-20x + 2x = -18x$

Putting everything together:

$$\mathbf{5x^2 - 18x - 8}$$

When you use variables instead of numbers, you might encounter a situation where you cannot add or subtract the outer and inner terms.

Example 2

For example,

$$\mathbf{(3a^2 + b)(a + 2c)}$$

Again, let's apply the FOIL method:

First: $(3a^2)(a) = 3a^3$

Outer: $(3a^2)(2c) = 6a^2c$

Inner: $(b)(a) = ab$

Last: $(b)(2c) = 2bc$

Putting everything together:

$$\mathbf{3a^3 + 6a^2c + ab + 2bc}$$

As we can see from the FOIL process, the variables for the outer and inner terms are different. In order to perform the addition or subtraction, the terms have to be the same.

Here is an example of similar terms being added or subtracted:

$$6a^2c + 3a^2c = 9a^2c$$

$$8ab - 2ab = 6ab$$

However, you can multiply and combine different terms together:

$$(6a^2c)(ab) = 6a^3bc$$

In the example here, you use the exponent property for the "a" variable:

$$(a^x)(a^y) = a^{x+y}$$

$$(a^2)(a^1) = a^{2+1} = a^3$$

Reversing FOIL: Un-FOILing

Now that we reviewed FOIL, we will look at how to reverse it through un-FOILing.

If we are given a quadratic polynomial, we can un-FOIL it by factoring it out as a product of two binomials.

Example 3

For example:

$$\boldsymbol{x^2 - 5x - 14}$$

Step 1: Examine the coefficients using the standard form of a quadratic function.

Recall that the standard form of a quadratic function is

$$\boldsymbol{ax^2 + bx + c}$$

In our example, a = 1, b = -5, and c = -14

Step 2: Examine the first term a.

In our example, a = 1. The value of "a" will equal the product of the "first terms" in the binomial. Since a = 1, we know that the two terms being multiplied will be (x) and (x):

$$(x)(x)$$

At this point we don't know which signs to put after the x. Thus, we continue further.

Step 3: Examine the final term c.

The c term corresponds to the "L" or last term in the binomial. In our example, it is -14.

Now we determine which two values when multiplied together will give us 14 (you can ignore the negative for now since we will consider the negative after choosing the product of the values).

The valid values when multiplied together (i.e. the factors) to give 14 are the following

1 and 14

2 and 7

Since the c term is negative, we note that one value is negative and the other is positive. For now we keep all of this in mind and move to the b term.

Step 4: Examine the middle term b.

The b term in our example is -5. Since the b term corresponds to the outer and inner terms added together in the FOIL method, we have to look at which values in Step 3 will yield -5.

The only possible combination is 2 and 7.

Next, we note the sign of b which is a negative. Since b = -5, we note that the larger value has to be negative. Thus, the combination will be 2 and -7.

Step 5: Use the previous information to factor out the polynomial and verify the result through FOIL.

$$\boldsymbol{(x+2)(x-7)}$$

Now that we factored out the polynomial, let's apply FOIL to verify:

First: $(x)(x) = x^2$

Outer: $(x)(-7) = -7x$

Inner: $(2)(x) = 2x$

Last: $(2)(-7) = -14$

This gives us:

$$\boldsymbol{x^2 - 5x - 14}$$

Now let's take a look at different situations which you may encounter when un-FOILing a quadratic polynomial. These situations apply when "a" = 1. If "a" does not equal one, it becomes a little more involved. We will discuss this after we get through this section.

<u>The sign of c</u>

One situation we want to also examine is Step 3 from our previous example. This deals with the sign of the two values being multiplied to yield "c".

If the sign of c is <u>negative</u>, one value is negative while the other is positive.

If the sign of c is <u>positive</u>, you may encounter one of the following:

- **Both factor values are positive**
- **Both factor values are negative**

Recall that the product of two positive values is positive. Similarly, the product of two negative values is positive.

Situation 1: Both are positive values

For example,

$$(x^2 + 9x + 8)$$

The sign of c is positive: 8

When we factor out the polynomial:

$$(x + 1)(x + 8)$$

Both values for the factors are positive.

Situation 2: Both are negative values

Another scenario you may encounter is the following:

$$(x^2 - 9x + 8)$$

Similar to the previous example, the sign of c is positive: 8
However, this time the sign for the "b" term is negative.

When we factor the polynomial out,

$$(x - 1)(x - 8)$$

Both values for the factors are negative.

When the factors being multiplied to yield "c" have the same sign, we note the following:

If both signs for the two factors being multiplied together for "c" are positive, the sign of b will be positive. (Situation 1)

If both signs for the two factors being multiplied together for "c" are negative, the sign of b will be negative. (Situation 2)

<u>*The sign of b*</u>

Another situation we want to examine is Step 4 from our previous example. This deals with the sign of the two factors being added to yield "b". This situation occurs when the factors have different signs for "c" (i.e. the sign for "c" is negative).

If the sign of b is negative, the greater value is negative while the lesser value is positive.

If the sign of b is positive, the greater value is positive while the lesser value is negative.

Situation 1: Negative c term, Negative b term

$$x^2 - 6x - 16$$

$$(x - 8)(x + 2)$$

Situation 2: Negative c term, Positive b term

$$x^2 + x - 20$$

$$(x + 5)(x - 4)$$

The value of "a"

So far our FOIL examples have been simple where a = 1. But what if "a" does not equal 1?

In our first situation, one of the factors being multiplied is 1 while the other is not 1.

In the second situation, both factors involved do not equal 1.

We will also consider the value for "c" which affects the signs for the two linear factors.

Let's take a look at examples of both situations.

Situation 1a:
"a" value involving 1 and another number not equal to 1
"c" value is positive

For example,

$$\mathbf{5x^2 - 13x + 6}$$

Again, we will begin by following similar steps to the previous example.

Step 1: Examine the coefficients using the standard form of a quadratic function.

a = 5

b = -13

c = 6

Step 2: Examine the first term a.

Note which factors when multiplied together will give the value for "a". In our case, 1 and 5 are the only candidates that can give five when multiplied together.

Unlike the previous section where there can only be one case for “a”, we now have two cases to consider:

$$(5x \pm \_\_)(x \pm \_\_)$$

or

$$(x \pm \_\_)(5x \pm \_\_)$$

As we can see, in both case multiplying the first terms will yield the same result: $5x^2$

The placement of “a” matters because it can affect the values for the outer and inner terms. This is usually not a problem if you put the correct factor in the correct position since you will get the same answer either way. For this section, it won’t matter which of the two you use from above since the signs are the same.

However, it gets a little trickier later on when the signs of the linear factors are different as well as the factors multiplied for the “First” term do not equal one. In that case, there will be a bit of adjusting and juggling. We’ll get to that later on.

Step 3: Examine the last term c.

Recall that the c term corresponds to the “L” or last term multiplied in the binomials. In our example, it is 6.

Now we determine which two values when multiplied together will give us 6.

The valid values when multiplied together (i.e. the factors) to give 6 are the following pairs:

1 and 6

2 and 3

Since the c term is positive, we note that both values are either negative or both values are positive. For now we keep all of this in mind and move to the b term.

Step 4: Using the information from “a” and “c” determine a combination that will yield the “b” term.

Unlike the previous example where a = 1, we now have to consider the factors for “a” and the factors for “c” together to determine “b”.

However, using the information from “c”, we know that both factor values have the same sign to yield c:

For the positive signs:

$(5x + \_\_)(x + \_\_)$

or

$(x + \_\_)(5x + \_\_)$

For the negative signs:

$(5x - \_\_)(x - \_\_)$

or

$(x - \_\_)(5x - \_\_)$

Since the b term is negative (-13) and the c term is positive, we will use the negative signs.

After writing down the possibilities, test the different combinations.

Step 5: Test the different possibilities to yield the "b" term.

Using the information from the previous four steps, we can test the different factors.

We know that the two factors have the same sign to yield "c".

We know that "b" is negative which means that the factors multiplied to yield "c" are both negative.

From looking at our options, we do not want $5x$ to be multiplied by 6 since that would yield $-30x$ while the other value will be $-1x$. Adding them together would equal $-31x$ and would be far from -13.

Similarly, if 6 was multiplied by x then $5x$ would be multiplied by 1 which would give $-11x$.

Thus, 1 and 6 will be ruled out. This will leave us with 2 and 3.

Next, we will use 2 and 3. If we multiply $5x$ by 3, we will get $-15x$. Then the x term will be multiplied by 2. This will yield $-17x$ for the b term.

So through process of elimination, we note that $5x$ must be multiplied by 2 and x must be multiplied by 3. We also note that since "b" is negative, we use the negative linear factors from Step 4. Since the signs are the same (both negative), it doesn't matter which of the linear factors you use from Step 4. The only thing that matters is that $5x$ is multiplied by 2 and x is multiplied by 3.

$$(5x-3)(x-2)$$

or

$$(x-2)(5x-3)$$

Although this one was a little easier, it gets trickier if the "c" term is negative since that will mean that the signs will be different. This will affect the outer and inner terms as we had discussed at the end of Step 2.

Before we continue, let's verify that the product of the two linear factors gives us the polynomial from the beginning of the section (you can use either from above):

$$(5x-3)(x-2)$$

First: $(5x)(x) = 5x^2$

Outer: $(5x)(-2) = -10x$

Inner: $(-3)(x) = -3x$

Last: $(-3)(-2) = 6$

or

$$(x-2)(5x-3)$$

First: $(5x)(x) = 5x^2$

Outer: $(x)(-3) = -3x$

Inner: $(-2)(5x) = -10x$

Last: $(-2)(-3) = 6$

$$\mathbf{5x^2 - 13x + 6}$$

Situation 1b:
"a" value involving 1 and another number not equal to 1
"c" value is negative

Now we will consider the approach when the "c" term is negative. This will mean that the two factors being multiplied to yield "c" have different signs.

For example,

$$\mathbf{3x^2 - 10x - 8}$$

So our final solution should look like the following:

$$(\_\_ + \_\_)(\_\_ - \_\_)$$

or

$$(\_\_ - \_\_)(\_\_ + \_\_)$$

Again, it is important to note that you can get the solution from either form. The only thing that matters is that the correct factors are in the correct position when performing the multiplication.

Let's go through each step carefully. The instructions are the same as the previous example we did from Section 1a. When there are differences, I will note them.

Step 1: Examine the coefficients using the standard form of a quadratic function.

a = 3

b = -10

c = -8

Step 2: Examine the first term "a".

Using the information from "a" we have two choices:

$$(3x + \_\_)(x - \_\_)$$

or

$$(3x - \_\_)(x + \_\_)$$

Now we have to consider the signs because you will get a different answer depending on which of the above you use. The most important things are that the factors in the blanks are put in the correct position (i.e. the correct values are attained when performing FOIL) and the signs for each coefficient are correct.

Step 3: Examine the last term "c".

The c term is negative which means that the signs will differ when multiplying the two factors. The factors for 8 are the following:

1 and 8

2 and 4

Step 4: Using the information from "a" and "c" determine a combination that will yield the "b" term. Test the different possibilities to yield a "b" term.

For $(3x + \_\_)(x - \_\_)$,

The first thing we always look at is the sign for "b". Since "b" is negative, the larger/greater value is negative when multiplied together while the smaller/lesser is positive.

If 8 is multiplied by $3x$, then x is multiplied by 1. The result will yield $-23x$.

If 1 is multiplied by $3x$, then x is multiplied by 8. The result will yield $5x$. (Also note that since the larger value has to be negative, this cannot be an option.)

Thus, 2 and 4 are our next candidates.

If 4 is multiplied by $3x$, then x is multiplied by 2. The result will yield $-10x$.

Again, if 2 is multiplied by $3x$, then x is multiplied by 4. The result will yield $-2x$.

For $(3x - \_\_)(x + \_\_)$,

If 8 is multiplied by $3x$, then x is multiplied by 1. The result will yield $23x$.

If 1 is multiplied by $3x$, then x is multiplied by 8. The result will yield $-5x$.

Thus, 2 and 4 are our next candidates.

If 4 is multiplied by $3x$, then x is multiplied by 2. The result will yield $10x$.

Again, if 2 is multiplied by $3x$, then x is multiplied by 4. The result will yield $2x$.

As we can see, the signs are reversed. If you are not careful, you may have the correct numbers but the incorrect signs.

We then get our correct combination:

$$(3x + 2)(x - 4)$$

or

$$(x - 4)(3x + 2)$$

Step 5: Perform FOIL to verify your result.

$$(3x + 2)(x - 4)$$

First: $(3x)(x) = 3x^2$

Outer: $(3x)(-4) = -12x$

Inner: $(2)(x) = 2x$

Last: $(2)(-4) = -8$

or

$$(x - 4)(3x + 2)$$

First: $(x)(3x) = 3x^2$

Outer: $(x)(2) = 2x$

Inner: $(-4)(3x) = -12x$

Last: $(-4)(2) = -8$

$$\mathbf{3x^2 - 10x - 8}$$

If you notice that the numbers are correct but the signs are switched you will get something like this:

$$(3x - 2)(x + 4)$$

$$3x^2 + 10x - 8$$

In this case, switch the signs without moving the numbers and test again. It should give you the correct result.

Situation 2a:
both "a" value factors are not equal to 1
"c" value is positive

As we can see, each situation becomes more complex as we consider different factors to yield the specified coefficient.

In the beginning, the coefficient for "a" was 1.

Next, the coefficient was a product of 1 and a value other than 1.

Now, we will look at the coefficient for "a" where both factors are not 1.

For example,

$$28x^2 + 29x + 6$$

Step 1: Examine the coefficients using the standard form of the quadratic equation.

a = 28

b = 29

c = 6

Step 2: Examine the first term "a".

Here is where we diverge a bit from the previous two sections. We now have to consider the factors for "a".

The factors for 28 are the following:

1 and 28

2 and 14

4 and 7

These pairs will yield 28.

Step 3: Examine the last term "c".

The last term, c, equals 6. Since it is positive, the signs for the two factors must be the same. However, since the "b" term is positive, the signs for the two factors being multiplied must be positive.

Next we note the factors of 6 and the product of the pairs to yield it:

1 and 6

2 and 3

Step 4: Test the combinations for "a" and "c" which will yield "b".

This is where it gets tricky. We now have to consider the factors for "a" along with the factors for "c" to get the correct value for "b". The one thing we do know from the previous two steps is that the signs are positive for the "c" term factors because "b" is positive.

$$(\_\_x + \_\_)(\_\_x + \_\_)$$

From here onward, it is a bit of trial and error. However, we can begin with the "c" term since we only have to test two pairs. The similar signs also make things a bit easier (as we had seen in the previous examples).

$$(\_\_x + 1)(\_\_x + 6)$$

or

$$(\_\_x + 2)(\_\_x + 3)$$

For $(\_\_\boldsymbol{x} + \mathbf{1})(\_\_\boldsymbol{x} + \mathbf{6})$,

if we use 1 and 28, the results for the "b" term are $34x$ or $169x$.

if we use 2 and 14, the results for the "b" term are $26x$ or $86x$.

if we use 4 and 7, the results for the "b" term are $31x$ or $46x$.

Thus, we have eliminated 1 and 6 as the factors to be used for "c". This will leave us with 2 and 3 as the correct factors for "c".

For $(\_\_\boldsymbol{x} + \mathbf{2})(\_\_\boldsymbol{x} + \mathbf{3})$,

if we use 1 and 28, the results for the "b" term are $59x$ or $86x$.

if we use 2 and 14, the results for the “b” term are 34*x* or 46*x*.

if we use 4 and 7, the results for the “b” term are 26*x* or 29*x*.

As we can see 4 and 7 are the correct factors to use for “a”. We can also see that the position matters so that we get the correct “b” term when adding the products of the outer and inner terms.

$$(4x+2)(7x+3) = 28x^2 + 26x + 6$$

$$(7x+2)(4x+3) = 28x^2 + 29x + 6$$

Thus, the second result is what will give us the correct polynomial.

Situation 2b:
both “a” value factors are not equal to 1
“c” value is negative

This will be the final section regarding the “a” term. It is also the trickiest one since the signs of the factors for “c” are different.

For example,

$$27x^2 + 60x - 32$$

Step 1: Examine the coefficients using the standard form of the quadratic equation.

a = 27

b = 60

c = -32

Step 2: Examine the first term “a”.

The factors for 27 are the following:

1 and 27

3 and 9

These pairs will yield 27.

Step 3: Examine the last term “c”.

The last term, c, equals -32. Since it is negative, the signs for the two factors must be different. Since the “b” term is positive, the greater value of the product must be positive while the lesser value of the product must be negative when adding the outer and inner terms.

Next we note the factors of -32 and the product of the pairs to yield it:

1 and 32

2 and 16

4 and 8

This time the “a” terms have two pairs and the “c” terms have three pairs to test for. Since “a” will be easier to work with, we test with “a” first.

Step 4: Test the combinations for “a” and “c” which will yield “b”.

For $(x + \_\_)(27x - \_\_)$,

if we use 1 and 32, the results for the “b” term are -5x or 863x.

if we use 2 and 16, the results for the “b” term are 38x or 430x.

if we use 4 and 8, the results for the “b” term are 100x or 212x.

Since none of these yield the correct term for “b”, we know that the correct factors for “a” are 3 and 9.

For $(9x + \_\_)(3x - \_\_)$,

if we use 1 and 32, the results for the “b” term are -285x or 87x.

if we use 2 and 16, the results for the “b” term are -138x or 30x.

if we use 4 and 8, the results for the “b” term are -60x or -12x.

As we can see from the final test, we do have the value similar to “b” but the sign is negative instead of positive: $(9x + 4)(3x - 8)$

When this happens, just switch the signs of the factors of the “c” terms and test again.

$$(9x - 4)(3x + 8)$$

$$\mathbf{27x^2 + 60x - 32}$$

Factoring Cubic Polynomials

Part I: Grouping and a Common Factor

When we are given a cubic polynomial in the standard form:

$$\boldsymbol{ax^3 + bx^2 + cx + d = 0}$$

it might not be as simple to factor it out.

Example 4

Let's begin with an easy example. For example,

$$\boldsymbol{x^3 - 2x^2 + 7x - 14 = 0}$$

Step 1: Put the cubic polynomial in standard form.

In our example, the cubic polynomial is already in standard form (meaning that it is arranged from highest to lowest exponent). If it isn't, rearrange it first.

Step 2: Group the first two terms together and the last two terms together.

$$[(x^3 - 2x^2)] + [(7x - 14)]$$

Step 3: Identify the greatest common factor (GCF) in the group and factor it out.

The greatest common factor is the "greatest" or highest factor that can divide the two terms in the group.

For the first group, the GCF is x^2. Thus, we factor it out and get

$$x^2(x - 2)$$

For the second group, the GCF is 7 since both numbers are divisible by 7. Thus, we factor it out and get

$$7(x-2)$$

Putting them back together we get

$$x^2(x-2)+7(x-2)=0$$

Step 4: Identify the common factor between the two groups and factor it out.

After factoring out the GCF, we now focus on the common factor between the two groups.

As we can see from Step 3, x^2 and 7 both have a common factor: (x-2)

We factor out (x-2) which gives us

$$(x-2)[x^2+7]=0$$

You can use the FOIL method to verify the result with the starting polynomial:

First: $(x)(x^2) = x^3$

Outer: $(x)(7) = 7x$

Inner: $(-2)(x^2) = -2x^2$

Last: $(-2)(7) = -14$

Putting it together we get

$$x^3-2x^2+7x-14$$

which verifies our answer.

Part II: Without a Common Factor

The previous example was simple since after grouping and factoring out the GCF, we found a common factor. However, this will not be the case in many situations.

For example,

$$\mathbf{2x^3 - 6x^2 + x + 9 = 0}$$

When we perform the process similar to Part I by grouping and then factoring out the GCF, we get the following

$$2x^2(x-3) + (x+9) = 0$$

Unlike Part I, we do not have a common factor that we can factor out from the two groups. Thus, we will not be able to proceed with Step 4 for this example. In this situation, we will perform different steps in order to factor out the cubic polynomial.

Step 1: Identify the "d" in the polynomial along with its factors.

After arranging the cubic polynomial in standard form, examine the "d" which is the value which does not have a variable attached to it.

In our example, d = 9

After we have the value for "d", we have to identify its factors. The factors for 9 are 1, 3, and 9. Recall that factors are values which can divide the number evenly. 9 can divide evenly by 1, 3, and 9.

Although positive factors are usually discussed, we also have to consider the negative factors: -1, -3, and -9 for this step.

Step 2: Plug in the positive and negative factors into the polynomial until it equals zero.

In this Step we will plug in the values into the polynomial until it equals zero. Recall that the x-value which makes the polynomial equal zero is a solution.

For 1: $2(1)^3-6(1)^2+(1)+9 = 6$

For -1: $2(-1)^3-6(-1)^2+(-1)+9 = 0$

Once you have found the factor which makes the polynomial zero, you can continue forward. In our case, we identified x = -1 so we do not have to test anymore.

Step 3: Bring the factor to the same side as the *x*. Prepare for synthetic division.

$$x = -1$$

$$(x+1) = 0$$

With the linear factor $x + 1$, we can perform synthetic division with the original polynomial. Since $x + 1$ is a factor of the polynomial, we should get a zero remainder when we perform the division.

Step 4: Perform the synthetic division.

The linear factor will be the divisor and the polynomial will be the dividend.

$$\begin{array}{r l}
 & 2x^2 - 8x + 9 \\
\cline{2-2}
\mathbf{x + 1} \,\big) & 2x^3 - 6x^2 + x + 9 \\
 & -2x^3 - 2x^2 \\
\cline{2-2}
 & \quad -8x^2 + x \\
 & \quad 8x^2 + 8x \\
\cline{2-2}
 & \qquad 9x + 9 \\
 & \qquad -9x - 9 \\
\cline{2-2}
 & \qquad\quad \mathbf{0}
\end{array}$$

To perform synthetic division, begin by dividing the first term in the dividend by the first term in the divisor. In the example we divide $2x^3$ by x. This gives us $2x^2$. Put the result at the top of the dividend (the polynomial).

Next, multiply the term on top by the divisor. In the example, we multiply $2x^2$ by x and then perform the same step again with 1. This will give us $2x^3 + 2x^2$. Write the result below the dividend. Once you write it below the dividend, reverse the signs so that the left term cancels out.

Next, perform the operation for the right term and its corresponding variable in the dividend. In our case, we get $-6x^2 - 2x^2$ to give us $-8x^2$. Bring down the next term from the dividend. In our case it is x.

Next, we start from the beginning and perform the same steps. In this case, we divide $-8x^2$ by x. This will give us $-8x$. Then, we write the result on top.

Again, multiply the term on top by the divisor. Here we multiply $-8x$ by x and by 1. We put them below the previous result under the dividend and then reverse the signs.

Do the same procedure again for $9x + 9$.

As we had expected, we get a zero remainder.

Step 5: Verify the result through multiplication and/or simply the expression further.

$$(x+1)(2x^2-8x+9)$$

After performing the synthetic division, you can choose to simplify further if the quadratic polynomial allows it. In this example, $2x^2$-$8x$ +9 cannot be factored further so we just move to the verification.

In the final example we will do, we will look at simplifying further.

To perform the multiplication, start with the leftmost term in the first parentheses and multiply it with all of the terms in the second parentheses (i.e. multiply x by the right polynomial). Do the same with the next term (i.e. multiply 1 by the right polynomial).

Make sure to line up similar terms. Afterwards, add the lines together.

$$\begin{array}{rrrr}
2x^3 & -\,8x^2 & +\,9x & \\
 & 2x^2 & -\,8x & +\,9 \\
\hline
2x^3 & -\,6x^2 & +\,x & +\,9
\end{array}$$

As we can see, we end up with the same cubic polynomial from the beginning!

Let's do a final example to tie everything together.

Example 5

Factor the following cubic polynomial:

$$\boldsymbol{x^3+2x^2-13x+10=0}$$

As we can see, the polynomial cannot be grouped easily similar to the previous example. Thus, we apply the method that we had used in the previous example.

Step 1: Identify the "d" in the polynomial along with its factors.

The "d" in this polynomial is 10.

The factors are ± 1, 2, 5, and 10.

Step 2: Plug in the positive and negative factors into the polynomial until it equals zero.

If you graph the polynomial, you will see that several factors will make it equal 0. If you test all of the positive and negative factors, the following will make the polynomial equal zero:

- 5, 1, 2

For -5: $(-5)^3+2(-5)^2-13(-5)+10 = 0$

For 1: $(1)^3+2(1)^2-13(1)+10 = 0$

For 2: $(2)^3+2(2)^2-13(2)+10 = 0$

If you do all of the tests, you will factor out the cubic polynomial:

$$(x+5)(x-1)(x-2)$$

If you don't, it's fine because we will be able to factor it out completely using synthetic division.

Assuming that you only test one factor, let's say $x = 1$ we will be able to continue to the next step. (The same procedure will hold true if you use $x = 2$ or $x = -5$)

Step 3: Bring the factor to the same side as the *x*. Prepare for synthetic division.

$$x = 1$$

$$(x-1) = 0$$

With the linear factor $x - 1$, we can perform synthetic division with the original polynomial. Since $x - 1$ is a factor of the polynomial, we should get a zero remainder when we perform the division.

Step 4: Perform the synthetic division.

$$
\begin{array}{r|l}
 & x^2 + 3x - 10 \\
\hline
\mathbf{x - 1} & x^3 + 2x^2 - 13x + 10 \\
 & \underline{-x^3 + x^2} \\
 & \qquad 3x^2 - 13x \\
 & \qquad \underline{-3x^2 + 3x} \\
 & \qquad\qquad -10x + 10 \\
 & \qquad\qquad \underline{10x - 10} \\
 & \qquad\qquad\quad \mathbf{0}
\end{array}
$$

Step 5: Verify the result through multiplication and/or simply the expression further.

$$(x - 1)(x^2 + 3x - 10)$$

For this step, we can factor out the quadratic polynomial to simplify further.

Recall the standard form of a quadratic function:

$$\boldsymbol{ax^2 + bx + c}$$

Since there is no explicit coefficient written for “a”, we know that the first term (“F” term in the FOIL method) is a product of two x-values: $(x)(x) = x^2$

The next thing we do is look at the “c” term which corresponds to the “L” (the last term) when we use the FOIL method. In our problem, it is -10.

Next we look at the “b” term which corresponds to the addition of the outer and inner terms (“O” and “I”). In our problem, it is 3.

Next find out which combination of numbers when multiplied together will equal 10 (you can mentally add the negative sign to one of the values). The valid values are the following:

1 and 10

2 and 5

Since the value is negative for 10, one of the two values is negative while the other is positive.

Finally from the two groups recognize which combination when added together will give 3. The only possible choice is the 2 and 5. Since the sign of "b" is positive for the 3, we know that the larger value is positive and the smaller value is negative.

$$x^2 + 3x - 10 = (x + 5)(x - 2)$$

When you perform the FOIL method, it verifies our solution.

$$\boldsymbol{x^3 + 2x^2 - 13x + 10 = 0}$$

$$\boldsymbol{(x - 1)(x + 5)(x - 2) = 0}$$