Calculus 2 Review
Handout 1.1A (to accompany Handout 1.1)
Basic Integration Rules

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Basic Integration Rules

Antiderivative of Zero:

$$\int \mathbf{0}\, dx = C$$

where C is an arbitrary constant

Constant Multiple Rule:

$$\int kf(x)\, dx = k\int f(x)\, dx$$

where *k* is any real number

Power Rule for Integration

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C, \qquad n \neq -1$$

Special Case of the Power Rule:
in this special case where n = 0, the Power Rule implies that:

$$\int k\, dx = kx + C$$

where *k* is any real number

Sum and Difference Rules

$$\int [f(x) \pm g(x)]\, dx = \int f(x)\, dx \pm \int g(x)\, dx$$

Basic Integration Rules for Trigonometric Functions

$\int \cos x \; dx = \sin x + C$	$\int \csc x \cot x \; dx = -\csc x + C$
$\int \sin x \; dx = -\cos x + C$	$\int \sec x \tan x \; dx = \sec x + C$
$\int \sec^2 x \; dx = \tan x + C$	$\int \csc^2 x \; dx = -\cot \; x + C$

Integration Rules for Exponential Functions

$$\int e^x\, dx = e^x + C$$

$$\int a^x\, dx = \left(\frac{1}{\ln a}\right) a^x + C$$

$$= \left(\frac{a^x}{\ln a}\right) + C$$

Integration Rule for Natural Logarithms

$$\int \frac{1}{x} dx = \ln|x| + C$$

Recall from the Power Rule for Integration that n cannot equal -1 (that is n ≠ -1). When we want to evaluate $\int x^{-1}\, dx$, we have to use the natural logarithm rule. The absolute value of x ($|x|$) ensures that the value of x is defined for the domain of $\ln x$.

Remember, if x is less than or equal to zero (that is x ≤ 0), $\ln x$ is **undefined**.

We will revisit and expand on these basic integration rules when we review integration by substitution.

For example, with u-substitution

$$\int \frac{1}{u} du = \ln|u| + C$$

rewritten as

$$\int \frac{du}{u} = \ln|u| + C$$

where $du = u'dx$.

(Please see the differentials section in the accompanying Handout 1.1 for more information)

When we find the antiderivative for

$$\int \frac{2x+5}{x^2+5x} dx$$

$$u = x^2 + 5x$$
$$du = (2x+5)dx$$

$$\frac{du}{2x+5} = dx$$

We substitute the u in the rule to get our solution:

$$\int \frac{2x+5}{x^2+5x} dx = \ln|x^2+5x| + C$$

We'll go into more detail when we review substitution, but the steps for this are as follows:

$$\int \frac{2x+5}{u} \frac{du}{2x+5}$$

The 2x + 5 cancels out and simplifies to

$$\int \frac{1}{u} du$$

and from our integration rule for natural logarithms (which leads us to our solution):

$$\int \frac{1}{u} du = \ln|u| + C$$