

Calculus 2 Review Handout 1.1A (to accompany Handout 1.1) Basic Integration Rules

by Kevin M. Chevalier

le cahier

Basic Integration Rules

Antiderivative of Zero:

 $\int \mathbf{0} \, dx = C$

where C is an arbitrary constant

Constant Multiple Rule:

 $\int kf(x) \, dx = k \int f(x) \, dx$ where k is any real number

Power Rule for Integration

 $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \qquad n \neq -1$

Special Case of the Power Rule: in this special case where n = 0, the Power Rule implies that:

$$\int k\,dx = kx + C$$

where k is any real number

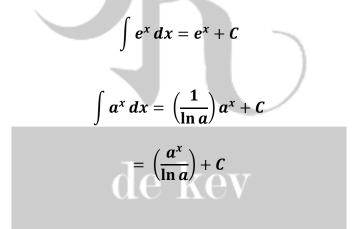
Sum and Difference Rules

$$\int [f(x) \pm g(x)] \, dx = \int f(x) \, dx \, \pm \, \int g(x) \, dx$$

Basic Integration Rules for Trigonometric Functions

$\int \cos x dx = \sin x + C$	$\int \csc x \cot x dx = -\csc x + C$
$\int \sin x dx = -\cos x + c \Theta C \partial \Omega$	$\int \sec x \tan x dx = \sec x + C$
$\int \sec^2 x dx = \tan x + C$	$\int \csc^2 x \ dx = -\cot x + C$

Integration Rules for Exponential Functions



Integration Rule for Natural Logarithms

$$\int \frac{1}{x} dx = \ln|x| + C$$

Recall from the Power Rule for Integration that n cannot equal -1 (that is $n \neq -1$). When we want to evaluate $\int x^{-1} dx$, we have to use the natural logarithm rule. The absolute value of x (|x|) ensures that the value of x is defined for the domain of $\ln x$.

Remember, if x is less than or equal to zero (that is $x \le 0$), $\ln x$ is **undefined**.

We will revisit and expand on these basic integration rules when we review integration by substitution.

For example, with u-substitution

$$\int \frac{1}{u} du = \ln|u| + C$$

rewritten as

 $\frac{du}{u} = \ln|u| + C$

where
$$\mathbf{d}\boldsymbol{u} = \boldsymbol{u}'\boldsymbol{d}\boldsymbol{x}$$

(Please see the differentials section in the accompanying Handout 1.1 for more information)

When we find the antiderivative for

$$\int \frac{2x+5}{x^2+5x} dx$$
$$u = x^2 + 5x$$
$$du = (2x+5)dx$$
$$\frac{du}{2x+5} = dx$$

We substitute the *u* in the rule to get our solution:

$$\int \frac{2x+5}{x^2+5x} dx = \ln|x^2+5x| + C$$

We'll go into more detail when we review substitution, but the steps for this are as follows:

$$\int \frac{2x+5}{u} \frac{du}{2x+5}$$

The 2x + 5 cancels out and simplifies to

$$\int \frac{1}{u} du$$

and from our integration rule for natural logarithms (which leads us to our solution):

$$\int \frac{1}{u} du = \ln|u| + C$$