Calculus Review
Handout 1.1 (Calc)
Graphs: Plotting, Intercepts, and Symmetry

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Topics: Graphing equations, finding intercepts of a graph, and symmetry

A graph helps us visualize its shape through methods such as point plotting. Given an equation, we can substitute a value for x to solve for y.

For example: $y = 2x + 4$

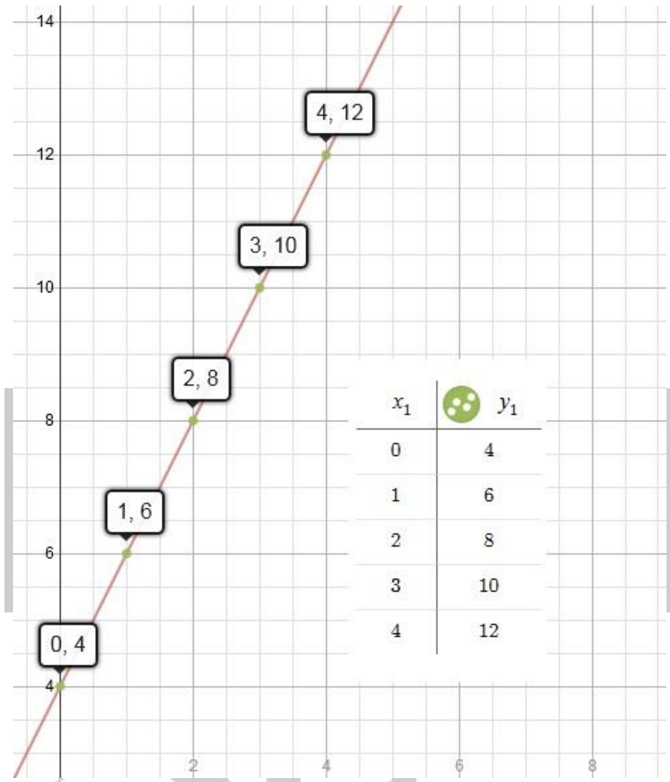
x	**0**	**1**	**2**	**3**	**4**
y	4	6	8	10	12

So for the first number, let $x = 0$ and solve for y.

$y = 2(0) + 4$
$y = 4$

The point (0, 4) is called a solution point since plugging in those numbers into the equation satisfies it. The same can be said for (1, 6), (2, 8), (3, 10), and (4, 12) when we repeat the process.

Next we plot the points from our table above to get an idea of the shape of the graph.



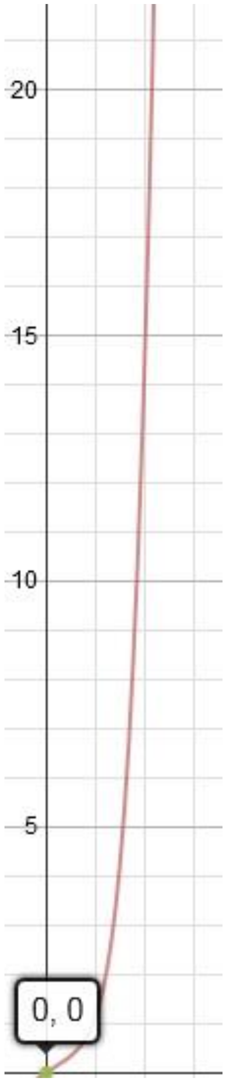
(Graphing capabilities thanks to Desmos.com)

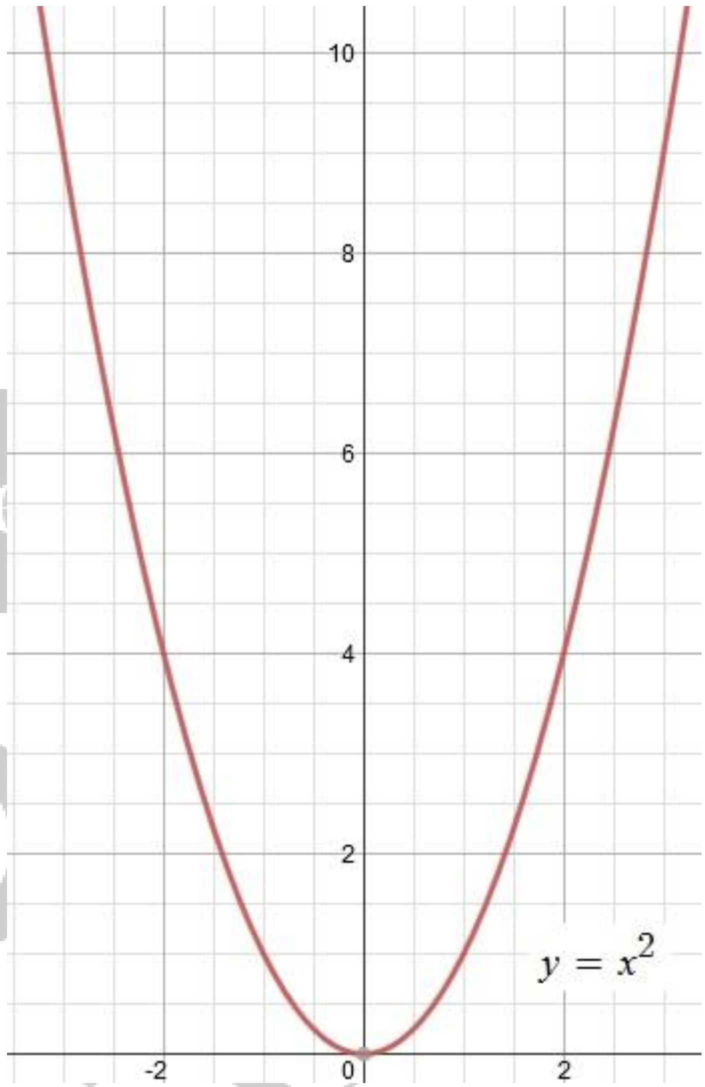
Now that we have plotted the points of the equation, we see that the graph is a straight line.

Note that plotting just a few points may not represent the true shape of the graph. We will see this later on when we graph polynomial equations.

For example, let's examine the equation: $y = \frac{1}{8}x(x^2 + 3x^4 + 5)$

We then plot nonnegative points to the right of the origin (0). We get this:

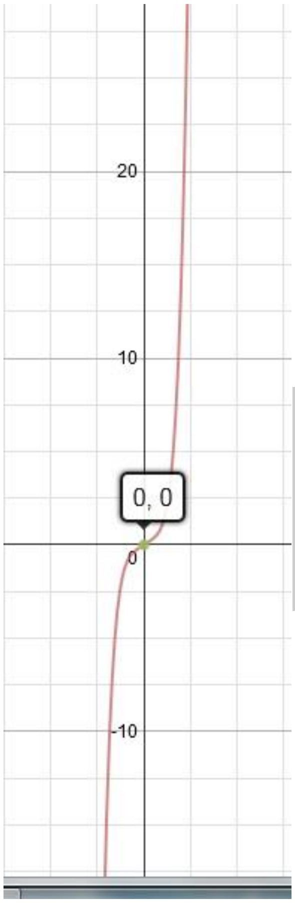




Our points plotted   A parabola

By looking at our graph from the points that we plotted at the origin and to the right of the origin, we will assume that the shape is similar to the parabola to the right. What usually happens is that people will quickly glance at the equation to determine its shape or just plot a few points and leave it at that.

However, when we plot the points to the left of the origin, we get a different picture:



From here we can see that we were wrong from assuming, and that the graph isn't a mirror image of the points that we would see in a parabola.

The graph is a little more complex than we thought!

On further examination, we see that we can multiply the $\frac{1}{8}x$ with the polynomial in parentheses and it will change the exponents. If you do that first, you will already know that the shape won't be a parabola.

Usually during exams or situations where you can't focus as easily, it will be easy to overlook these things. It's one thing when you're sitting down and you have the time to examine the equation while doing homework, but when you have to complete an exam in under an hour, it can be difficult.

The best way will be to recognize different shapes of graphs of different equations. Also, don't assume from plotting a few points that you have the graph's shape. In addition to quadratic ($y = ax^2 + bx + c$) and cubic functions ($y = ax^3 + bx^2 + cx + d$), we will see more shapes when we look at absolute values, square roots, and inverse functions (to name a few).

It's different!

I will cover the graphs of trigonometric functions in the precalculus review. If you're reading this worksheet, I'm assuming that you already have the foundations of precalculus under your belt. But you can always go back and review in case you're not sure!

Everyone has access to graphing software and capabilities now so if you're not sure, you can always double check.

Finding the Intercepts of a Graph

We previously discussed solutions points in the previous section where we plug in the numbers to satisfy the equation.

Intercepts are the points where the graph intersects the x-axis or y-axis.

To get the x-intercept (x, 0): let y = 0 and solve for x

To get the y-intercept (0, y): let x = 0 and solve for y

Let's do a simple quadratic equation. Remember the quadratic equation form:
$ax^2 + bx + c$

Example: Find the x- and y- intercepts for the graph of $y = x^2 - 9$.

x-intercept:

Let y = 0, so:
$x^2 - 9 = 0$

Step 1: Factor the equation.

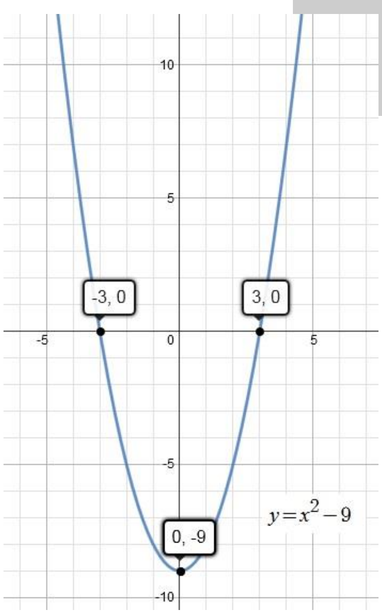
$(x + 3)(x - 3) = 0$

Step 2: Solve for x

For $(x + 3) = 0$, $x = -3$
For $(x - 3) = 0$, $x = 3$

Since we have two solutions, the graph has two x-intercepts: (-3, 0) and (3, 0)

y-intercept:



Let x = 0, so:
$y = (0)^2 - 9$

Solve the equation:
$y = -9$

The y-intercept is at (0, -9).

When we graph it, we can see the intercepts:

x-intercepts: (-3, 0) and (3, 0)
y-intercept: (0, -9)

Keep in mind that a graph may not have any intercepts or it may have many. In our example above, we had two x-intercepts and just one y-intercept.

Testing for a Graph's Symmetry

Y-axis symmetry

A graph is symmetric with respect to the y-axis if along with the point (x, y), (-x, y) is also a point on the graph.

We saw this in our example above for the equation $y = x^2 - 9$. We saw that the left and right sides of the graph (left of the y-axis and right of the y-axis, respectively) are a mirror image of each other.

To test for symmetry with respect to the y-axis, replace x with -x. If the final result is the same as your original equation, there is symmetry.

Example 1: Test the graph of $y = 4x + 2$ for symmetry with respect to the y-axis.

Step 1: Replace x with -x:

$y = 4(-x) + 2$
$y = -4x + 2$

Step 2: Match it to the original equation.

The final equation $y = -4x + 2$ is not equivalent to the original equation. There is no symmetry with respect to the y-axis.

Example 2: Test the graph of $y = |x| + 5$ for symmetry with respect to the y-axis.

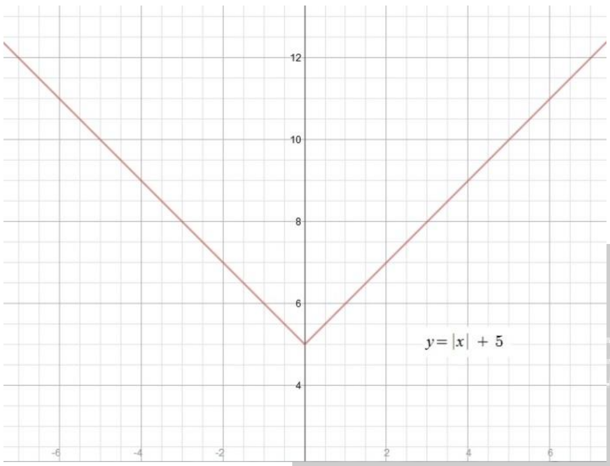
Step 1: Replace x with -x:

$y = |-x| + 5$
$y = x + 5$

*Remember absolute values are positive since they tell you the distance of x from zero, not which direction it is. If you have |x| (read as "the absolute value of x"), and whether we let x = 5 or x = -5, the value will always be 5 units from zero. So whether we put the positive or negative value of the number, they will always be x units from zero.

Step 2: Match it to the original equation.

Our solution: $y = x + 5$
Original equation: $y = x + 5$



The two equations are equivalent; therefore, there is symmetry with respect to the y-axis.

<u>X-axis symmetry</u>

A graph is symmetric with respect to the x-axis if along with the point (x, y), (x, -y) is also a point on the graph.

To test for symmetry with respect to the x-axis, replace y with -y. If the final result is the same as your original equation, there is symmetry.

We basically follow the same steps as we did with y-axis symmetry, but in this case, we replace y with -y to test for x-axis symmetry. The part above the x-axis should be a mirror image of the part below the x-axis.

Example 3: Test the graph of $y = x^3 + 2$ for symmetry with respect to the x-axis.

Step 1: Replace y with -y

$-y = x^3 + 2$

You can make y positive by multiplying everything by -1. You will get:

$y = - x^3 - 2$

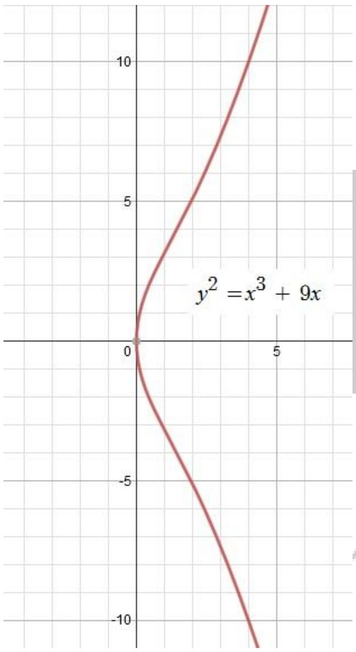
Step 2: Match it to the original equation.

In any case, the equations are not equivalent. There is no symmetry with respect to the x-axis.

Example 4: Test the graph of $y^2 = x^3 + 9x$ for symmetry with respect to the x-axis.

Step 1: Replace y with -y

$(-y)^2 = x^3 + 9x$
$y^2 = x^3 + 9x$



Step 2: Match it to the original equation.

Our solution: $y^2 = x^3 + 9x$
Original equation: $y^2 = x^3 + 9x$

The two equations are equivalent; therefore there is symmetry with respect to the x-axis.

<u>Symmetry with respect to the origin</u>

A graph is symmetric with respect to the origin if along with the point (x, y), (-x, -y) is also a point on the graph.

To test for symmetry with respect to the origin, replace x with -x and y with -y. If the final result is the same as your original equation, there is symmetry.

Origin symmetry means that if you rotate the graph by 180° about the origin, there should be no change.

I think I've already given examples of when the result does not match with the original equation for x- and y- axis symmetry so I'm going to jump right in with the example for origin symmetry.

Example 5: Test the graph of $y = \frac{1}{x}$ for symmetry with respect to the origin.

We follow the same steps as we did for testing x- and y- axis symmetry.

Step 1: Replace x with -x and y with -y.

(-y) = $\frac{1}{(-x)}$

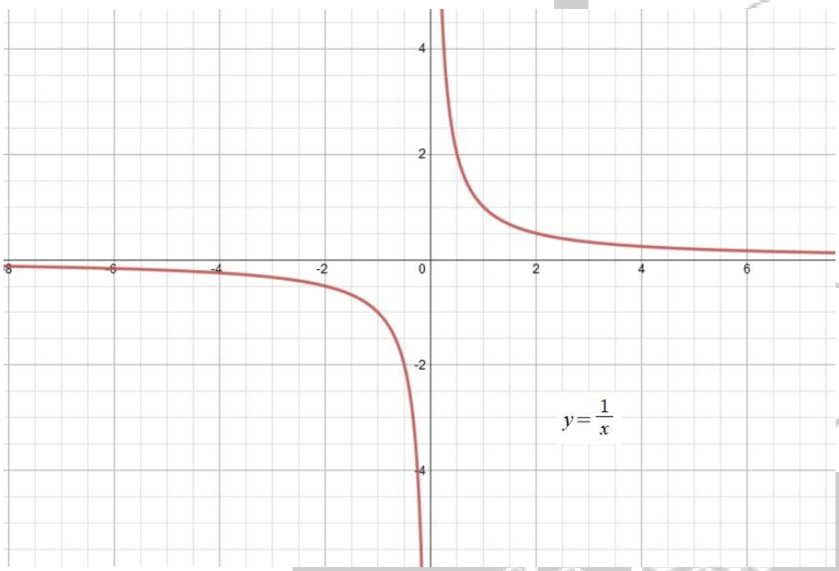
We then get:

-y = -$\frac{1}{x}$

We multiply both sides by -1 and we get our final answer:

y = $\frac{1}{x}$

The result and the original equation are both equivalent; therefore, we have symmetry with respect to the origin.



You can now use the intercepts and your new knowledge of symmetry to draw graphs. For example if we are sketching a graph that is symmetrical to the y-axis (such as a parabola), and we have the solution point (2 , 4), we now know that (-2, 4) is also a solution point.

Wow this was a lot to take in right? I'll stop here for now so that you can absorb it. But now you have the tools to graph the solution points faster.