Medical Mathematics
Handout 1.1
Review of Foundational Level Mathematics

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For many professionals in the health services industry, having a strong foundation in basic mathematical calculations is crucial. Examples include converting weight and temperature from one system to another (e.g. Fahrenheit to Celsius or pounds to kilograms) and calculating the appropriate dosages for a pediatric versus adult patient. However, all of these require a firm grasp of topics such as arithmetic and working with fractions and decimals.

Although this handout covers mathematical topics tailored to healthcare professionals, audiences from any background may find this information useful (especially if they just want to refresh their knowledge of foundational level mathematical concepts). Many of the topics covered here are definitely important for courses in higher level mathematics.

This handout will review order of operations, fractions, decimals, percentages, and proportions. We will also review concepts such as long division, long multiplication, and basic algebra (isolating the unknown variable to solve for its value). In the next handout, we will use this foundation to convert between different measurement systems.

Order of Operations

When you encounter a problem such as

$$\frac{4(5-3)^2+16}{8-4}$$

one of the questions that come to mind is "which operation should I perform first?"

PEMDAS is a common acronym that is taught to students to remember which operations to perform first. The standard version is "Please Excuse My Dear Aunt Sally" which stands for the order you need to perform:

Parentheses
Exponents
Multiplication
Division
Addition
Subtraction

I used something else: **P**lease **E**njoy **M**y **D**inner **A**t **S**chool. I loved trying new food and cooking back then (I still do!) so many of my mnemonics dealt with food! Choose whichever method works best for you.

After you perform the operations inside the parentheses and then the exponents, you evaluate the others (multiplication and division; addition and subtraction) from **left to right**.

Another important note is that when you have a fraction, you first simplify the numerator (top) and the denominator (bottom) and then perform the division. For example,

$$\frac{4(6-3)}{30+6}$$

We simplify 4(6-3) to get 12. Remember (6 - 3) comes first and then multiply that by 4. We then go to the denominator and simplify: 30 + 6 = 36. Now we can simplify and/or divide.

$$\frac{12}{36} = \frac{1}{3} \approx 0.\overline{3}$$

Let's see the order of operations in action and why it's important.

Example:

$24 \div 3(3\text{-}1)^2 - 4$

We perform what is inside the parentheses first:

$24 \div 3(\mathbf{3\text{-}1})^2 - 4$

$24 \div 3(\mathbf{2})^2 - 4$

Next we evaluate the exponents (in this case, it's cubed):

$(2)^{\mathbf{2}} = 4$

$24 \div 3(4) - 4$

Multiplication comes next (just multiply 3 by 4):

$$24 \div 3(4) - 4$$

$$24 \div 12 - 4$$

Now this is where some individuals make errors:

$$\mathbf{24 \div 12 - 4}$$

Under the rules for the order of operations, division comes first. It is incorrect to perform the subtraction first. We will see what happens to our answer when we perform the 12 - 4 first:

$$24 \div 12 - 4$$

$$24 \div 8 = 3 \times$$

Incorrect!

Going from left to right, performing the division first and then the subtraction, we get the correct answer:

$$24 \div 12 - 4$$

$$\mathbf{2 - 4 = -2} \checkmark$$

We get two **completely different** answers if we do the incorrect order.

Another error is just ignoring PEMDAS and performing the operations from left to right:

$$\mathbf{24 \div 3(3-1)^2 - 4}$$

$$\mathbf{8(3-1)^2 - 4}$$

$$\mathbf{8(4) - 4}$$

$$\mathbf{32 - 4 = 28} \times$$

Incorrect!

Now let's work on our example together using the rules for the order of operations:

Remember: As we had discussed earlier, we have to simplify the numerator and denominator separately and then we can perform the division. Let's start with the numerator (the top):

$$\frac{4(5-3)^2+16}{8-4}$$

Parentheses:

$$\frac{4(\mathbf{2})^2+16}{8-4}$$

Exponents:

$$\frac{4(\mathbf{4})+16}{8-4}$$

Multiplication:

$$\frac{\mathbf{16}+16}{8-4}$$

Addition:

$$\frac{\mathbf{32}}{8-4}$$

Now we simplify the denominator (the bottom) and perform the subtraction:

Subtraction:

$$\frac{32}{\mathbf{4}}$$

Now that the numerator and denominator are simplified, we can divide:

Division:

$$\frac{32}{4}=\mathbf{8}$$

Once you get the hang of it, performing the order of operations will be intuitive/second nature to you.

<u>Fractions</u>

Fractions are expressions representing parts of a whole. Think of it as dividing something whole into parts or portions (e.g. a whole pizza consisting of eight slices (portions)).

The top part is the **numerator**. It represents the specified number of equal <u>parts</u> or <u>portions</u> of the whole.

The bottom part is the **denominator**. It represents the <u>total number</u> of parts that the whole consists of.

In this example,

$$\frac{\mathbf{3}}{\mathbf{8}}$$

The numerator is 3 (we are specifying 3 equal parts from the whole). The denominator is 8 (we are noting that the whole consists of 8 equal parts).

Going back to our example of an entire pizza (or an entire pie) and applying it here, the total number of slices is 8 and the number of slices we pick out are 3. If I ate 3 slices of pizza, I ate 3/8 of the whole pizza.

There are several types of fractions which we will discuss:

1. Proper Fractions
2. Improper Fractions
3. Mixed Fractions
4. Complex Fractions

<u>Proper Fractions</u>

In a **proper fraction**, the value of the numerator is smaller (less) than the denominator. **Since the value of the parts (the numerator) is smaller than the whole (the denominator), the value of a proper fraction is less than one whole unit.**

We will see in the next topic (improper fractions) that when the numerator and the denominator are equal to each other, the fraction equals 1.

For example,

$$\frac{2}{3} < \frac{3}{3}$$

When we convert it to decimals (just divide the numerator by the denominator-- in this case, 2 divided by 3),

$$\frac{2}{3} \approx 0.67$$

$$\frac{3}{3} = 1$$

$$0.67 < 1$$

We will go into more detail on converting fractions to decimals when we get to the section on decimal fractions.

<u>Improper Fractions</u>

In an **improper fraction**, the numerator is greater than or equal to the denominator.

For example, the numerator and denominator are equal to each other here:

$$\frac{5}{5} = 1$$

When the numerator and the denominator are equal, the value is always 1.

When you have a whole number, its denominator is 1 in fraction form:

$$10 = \frac{10}{1}$$

$$6 = \frac{6}{1}$$

In the next example, the numerator is greater than the denominator:

$$\frac{7}{4} \; or \; \frac{15}{8}$$

<u>Mixed Fractions</u>

A **mixed fraction (or mixed number)** consists of a whole number and a proper fraction put together. **The value of a mixed number is greater than 1**.

In order to convert an improper fraction into a mixed number, you first divide the numerator by the denominator. In some cases, you will get a whole number:

$$\mathbf{\frac{21}{3} = 21 \div 3 = 7}$$

In other cases, you will get a whole number and a remainder:

$$\mathbf{\frac{25}{4}}$$

Unlike the previous example, this one will not divide evenly (i.e. have a remainder of zero). When we perform the division, we get 6 with a remainder of 1.

The whole number multiplied by the denominator should be close to the value of the numerator without going over. So, in our example 6 x 4 = 24. Six is the closest whole number value to 25 without going over when we divide. When we use the next value (7) and multiply it by 4 (i.e. 7 x 4), we get 28. This value is greater than the numerator so we know that the value has to be less than (but close to) that.

Next, we have to find the remainder. To do that, subtract the value you got when you multiplied the whole number by the denominator (in our example it was 6 x 4 = 24) from the value of the numerator. This will give you the remainder.

25 - 24 = 1

Our remainder is 1. The remainder is the number that goes in the place of the numerator in the proper fraction of the mixed number.

Another point that I want to touch upon is that the value of the whole number should not be lower than the closest value. For example, if we chose 5 and multiplied 5 by 4 (5 x 4) we get 20. The remainder will then be 25 - 20 = 5. This will give us an improper fraction: $\frac{5}{4}$ and we know that a proper fraction has to combine with the whole number.

So keep in mind that if the remainder is greater than the denominator, you have to go back and see if you can divide the numerator by the denominator and get a remainder that is smaller than the denominator.

The denominator of the proper fraction of the mixed number is the same as the denominator of the improper fraction. The denominator of the improper fraction is 4 so we will also use this for the denominator of the proper fraction of the mixed number.

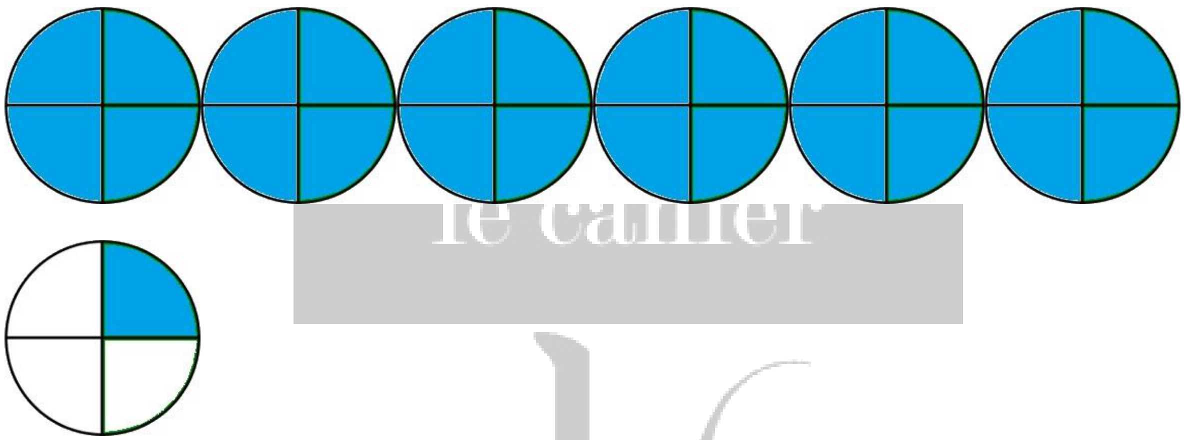
We now have all of the information we need to write our mixed number.

Whole number: 6

Numerator (Remainder): 1
Denominator: 4

$$6\frac{1}{4}$$

So in this example, we get 6 wholes with 4 complete parts and a remainder of 1 part:



Let's do another example to apply what we learned. After this example, we will look at how to convert a mixed number into an improper fraction.

Example: Convert $\frac{35}{3}$ into a mixed number.

Let's follow the same steps as we had done during the explanation.

Step 1: Divide the numerator by the denominator to find the whole number.

When we divide 35 by 3, the closest whole number we get without going over the number in the numerator is 11. We verify this by multiplying 3 by 11 which gives us 33. If we multiply by 12 (i.e. 3 x 12), we get 36 which is greater than the number of the numerator in the improper fraction. Also, if we choose something less (i.e. 3 x 10), we get 30 and when we subtract it from the original numerator (35 - 30) we get a remainder of 5 which is greater than the denominator (so we know it's wrong).

Step 2: Find the remainder.

When we subtract 33 from 35, we get 2. The remainder in fraction form is the subtracted value in the numerator and the original denominator in the denominator position.

$$\frac{2}{3}$$

Step 3: Combine the whole number and the proper fraction.

Whole number: 11
Remainder: 2
Denominator: 3

$$11\frac{2}{3}$$

<u>Converting Whole and Mixed Numbers to Improper Fractions</u>

Converting a whole number to an improper fraction is simple. Simply multiply the whole number with your chosen denominator.

For example, I want to convert 15 into a fraction with a denominator of 4:

$$\frac{?}{4} = 15$$

We just need to multiply 15 by 4 which gives us 60:

$$\frac{60}{4} = 15$$

This technique is important since we will use this when finding common denominators in order to compare fractions (i.e. which one is greater or less than the other) as well as perform operations such as adding or subtracting fractions with different denominators.

To convert a mixed number to an improper fraction, we make a "C" shape starting with the denominator.

Let's look at the following example:

$$4\frac{1}{3}$$

Using our "C" shape guide, we start with the denominator, 3. You multiply the denominator by the whole number.

$$3 \times 4 = 12$$

You then take that value and add it to the numerator:

$$12 + 1 = 13$$

Now that we have completed the “C”, we put that value in the numerator and use the same denominator from the improper fraction.

$$\mathbf{\frac{13}{3}}$$

We will be using fractions when looking at drug dosage calculations. **For example, if a hospital has 50 mg of scored (can be safely cut in half for an appropriate dose) Demerol tablets available and the physician orders a dose of 75 mg PO (by mouth) for an adult patient, how many tablets are in one dose?**

Using known information, the scored Demerol tablets are 50 mg. How many tablets will equal 75 mg? This will be our unknown labeled as *x*. Multiplying 50 mg by *x* (the number of tablets) signifies how many tablets times the strength of 50 mg will equal 75 mg.

$$50x = 75 \text{ mg}$$

We divide both sides by 50 to isolate the *x* on one side:

$$x = \frac{75}{50}$$

Our unknown is an improper fraction and we have to simplify it. You can use a calculator to quickly get the answer, but we want to use the methods that we learned to turn it into a mixed number.

The closest whole number when dividing the numerator by the denominator is 1. The remainder is 25.

$$1\frac{25}{50}$$

Since the proper fraction portion can be further simplified, we simplify it to:

Remember to simplify the fraction until the numerator and denominator cannot be divided by a common number. In our example, 25 and 50 can be divided by 25 evenly:

$$\frac{25}{50} = \frac{1}{2}$$

Put everything together:

$$\mathbf{1\frac{1}{2}}$$

So one dose for the adult patient is one and a half tablets.

Simplifying Fractions

In the previous example, we talked about simplifying fractions. Let's say we have the following:

$$\mathbf{\frac{30}{42}}$$

We can check to simplify this further into lowest terms by dividing the numerator and the denominator by a similar value that can yield a whole number. The best thing to do is find the highest number they both can evenly divide into. In this case it's 6.

$$\frac{30}{6} = 5$$

$$\frac{42}{6} = 7$$

$$\frac{30}{42} = \frac{5}{7}$$

We cannot simplify any further since we cannot divide the numerator and denominator by a common value.

What happens if you choose a value that is not the highest number they both can evenly divide into? It's no problem at all. You can just keep simplifying by dividing until you cannot divide the numerator and denominator by the same number. For example, let's use 3 instead of 6.

$$\frac{30}{42}$$

$$\frac{30}{3} = 10$$

$$\frac{42}{3} = 14$$

$$\frac{30}{42} = \frac{10}{14}$$

As we can see from the example, the fraction can still be simplified further since the numerator and denominator can be divided by 2.

$$\frac{10}{2} = 5$$

$$\frac{14}{2} = 7$$

$$\frac{30}{42} = \frac{10}{14} = \frac{5}{7}$$

In the end you still get the same answer. So don't worry if you don't choose the highest number to divide by right away. Further simplification will give you the correct answer in the end.

Complex Fractions

Complex fractions can either have a whole number, a proper fraction, or a mixed number in the numerator and/or denominator. For example:

$\frac{1/3}{3}$	$\frac{3/4}{1/2}$	$\frac{5\,3/4}{2/9}$

We will learn how to simplify complex fractions when we discuss dividing fractions.

Value of Fractions and Least Common Denominator

When comparing the value of two different fractions, we will use several methods to see which one is greater or less than the other.

If the fractions have a common denominator, look at the value of the numerator to determine which fraction has the smaller or larger value:

Example: Order the following fractions from lowest to highest value.

$$\frac{4}{9}, \frac{1}{9}, \frac{7}{9}, \frac{2}{9}$$

Since the denominators have the same value (9), we just order the fractions based on the numerator from lowest to highest value:

$$\frac{1}{9}, \frac{2}{9}, \frac{4}{9}, \frac{7}{9}$$

If the fractions have a common numerator, **the fraction with the smaller denominator has the higher value**.

Example: Which of the two has the higher value?

$$\frac{5}{7} \text{ or } \frac{5}{9}$$

Since 5/7 has the smaller denominator, it has the higher value. We can also verify this by finding the least common denominator and comparing the two fractions.

To get the least common denominator, you can multiply the denominators of the two fractions. (We will go into more detail when we discuss least common denominators in the following section). Going back to the example, 9 x 7 = 63. This will be the least common denominator between the two fractions since the denominators can be divided evenly with 63.

So then we get:

$$\frac{?}{63} \text{ or } \frac{?}{63}$$

Divide the least common denominator by the denominator of the fraction it is associated with. For the first one it is 63 ÷ 7. This equals 9. You take that number and multiply it by the associated numerator. For the first one it is 9 x 5. This equals 45. We then get:

$$\frac{45}{63}$$

Do the same for the second fraction. For the second one, 63 ÷ 9 = 7. Multiply that by the associated numerator. We then get 7 x 5 = 35. Thus, the fraction comes out to

$$\frac{35}{63}$$

We then look at our fractions side to side:

$$\frac{5}{7} \text{ or } \frac{5}{9}$$

$$\frac{45}{63} \text{ or } \frac{35}{63}$$

This verifies that 5/7 is greater than 5/9.

<u>Least Common Denominator</u>

We began discussing the least common denominator in the previous example. This concept is very important in mathematics.

When comparing fractions with different denominators, the simplest way is to find a common denominator for all of the fractions you are comparing.

To find the least common denominator, you have to find the smallest value for the denominator where all of the denominators in the original fractions can evenly divide with.

Example: Find the least common denominator of $\frac{1}{3}$, $\frac{7}{12}$, and $\frac{5}{6}$.

To begin, look at the denominators for all of the fractions. The number that comes to mind when checking which of the denominators is evenly divisible by the other denominators is 12.

12 is evenly divisible by 3 and 6 so this is the least common denominator.

If we want to find equivalent fractions using the least common denominator for all of the fractions, we apply what we did in the previous example.

First, divide 12 by each denominator:

12/3 = 4

12/ 6 = 2

Then, multiply that number by the value in the numerator:

For 1/3, we multiply 4 by 1 to get 4. This gives us

$$\frac{1}{3} = \frac{4}{12}$$

For 5/6, we multiply 2 by 5 to get 10. This gives us

$$\frac{5}{6} = \frac{10}{12}$$

Now that all of the fractions have the same denominator, we can order them from lowest to highest as well as perform operations such as addition and subtraction.

Similar to the example comparing 5/7 versus 5/9, if you cannot determine the least common denominator like what we did with 1/3, 5/6, and 7/12, just multiply the two denominators together. If this doesn't work, you will have to use trial and error. Don't worry because the fractions are usually easy to work with at this point.

Now that we reviewed fractions, let's apply them to the four basic operations of arithmetic: addition, subtraction, multiplication, and division.

<u>Addition and Subtraction with Fractions</u>

In order to add or subtract fractions with different denominators, you have to find the least common denominator.

You cannot do this:

$$\frac{7}{9} - \frac{2}{3} = \frac{5}{6}$$

Incorrect!

Let's use what we learned to solve this problem.

Example: Evaluate the following.

$$\mathbf{\frac{7}{9} - \frac{2}{3}}$$

$$\mathbf{\frac{3}{4} + \frac{1}{7}}$$

Let's start with the subtraction problem.

$$\frac{7}{9} - \frac{2}{3}$$

Looking at the two denominators, we can see that 9 is evenly divisible by 3.

$$\frac{2}{3} = \frac{6}{9}$$

9 ÷ 3 = 3. We then multiply this by the numerator (3 x 2 = 6).

Now we can perform the operation.

$$\frac{7}{9} - \frac{6}{9} = \mathbf{\frac{1}{9}}$$

After performing the operation, check to see if the answer can be reduced to lowest terms. In this case, we cannot simplify any further so this is our answer.

Let's work on the second one now.

$$\mathbf{\frac{3}{4} + \frac{1}{7}}$$

For this one, we can multiply the two denominators to get our least common denominator. (7 x 4 = 28) We then divide 28 by the denominators and multiply them with their respective numerators:

$$\mathbf{\frac{21}{28} + \frac{4}{28}}$$

We can perform the addition now.

$$\mathbf{\frac{21}{28} + \frac{4}{28} = \frac{25}{28}}$$

This cannot be reduced further so this is the final answer.

So far we looked at fractions. But how will we approach adding or subtracting fractions with whole numbers?

Example: Perform the indicated operation.

$$2 - \frac{3}{10}$$

When you have to find a common denominator for a fraction and whole number, simply multiply the whole number by the denominator of the fraction you are subtracting (or adding) with. In this case, 2 x 10 = 20. This will be the numerator for the whole number's fraction equivalent.

You then use the same denominator from the fraction. We then get

$$2 = \frac{20}{10}$$

$$\frac{20}{10} - \frac{3}{10} = \frac{17}{10}$$

We then simplify this:

$$\mathbf{\frac{17}{10} = 1\frac{7}{10}}$$

Multiplication with Fractions

Multiplying fractions is easier. Just multiply the numerators and then multiply the denominators to get your answer. You then reduce that to lowest terms.

Example: Evaluate the following.

$$\frac{3}{8} \times \frac{4}{5}$$

You just multiply the numerator and then multiply the denominator:

$$\frac{3}{8} \times \frac{4}{5} = \frac{12}{40}$$

Finally reduce the fraction to lowest terms (we can divide the numerator and the denominator by 4):

$$\frac{12}{40} = \frac{3}{10}$$

Our final answer is 3/10.

Another method is to check whether the numbers diagonal to each other can be simplified further before multiplying. In our example,

$$\frac{3}{8} \times \frac{4}{5}$$

We see that we cannot simplify 3 and 5 further. However, 4 and 8 can be simplified further since both numbers are divisible by 4. We then simplify where 4 ÷ 4 = 1 and 8 ÷ 4 = 2.

$$\frac{3}{2\,\cancel{8}} \times \frac{4\,1}{5}$$

You cross out the original numbers and place the simplified numbers next to it (the strikethrough is not easily visible on the 4, but it's there). You then multiply across:

$$\frac{3}{2} \times \frac{1}{5} = \mathbf{\frac{3}{10}}$$

We get the same answer! This shortcut is very useful since you save time by simplifying sooner than later (when the numerator and denominator become larger).

Division with Fractions

To divide two fractions, you flip the numerator and denominator of the divisor (the number which does the dividing). Basically speaking, the divisor is the one that comes after the division sign. After flipping the divisor, you perform multiplication to get your answer.

Example: Perform the following operation.

$$\frac{2}{9} \div \frac{1}{36}$$

For this example, the divisor is 1/36. We have to flip the numerator and denominator of the divisor and then multiply the fractions. We then get

$$\frac{2}{9} \times \frac{36}{1}$$

You have two choices here. First, you can perform the multiplication operation and then simplify.

$$\frac{2}{9} \times \frac{36}{1} = \frac{72}{9}$$

$$= 8$$

Another choice you can do is our diagonal shortcut to see if they can reduce further and then multiply. When we look at the numbers diagonal to each other, 2 and 1 cannot be simplified further. However, 9 and 36 can be reduced further by dividing both by 9. We then multiply. This will give

$$\frac{2}{1\,\cancel{9}} \times \frac{\cancel{36}\,4}{1}$$

$$\frac{2}{1} \times \frac{4}{1} = \frac{8}{1}$$

$$= 8$$

Either way is fine. Choose whichever works best for you. As we had discussed before, it's better to simplify sooner than later because the values of the numerator and denominator are larger after performing the multiplication without simplifying first.

Earlier on we discussed complex fractions. Let's use our knowledge of dividing fractions to simplify complex fractions.

Example: Simplify the following complex fractions.

$\frac{1/8}{6}$	$\frac{2/15}{4/5}$

We perform the same steps as we had done. First, rewrite the complex fraction using division terms.

For the first one we say "1/8 divided by 6."

$$\frac{1}{8} \div \frac{6}{1}$$

We flip the divisor and then multiply:

$$\frac{1}{8} \times \frac{1}{6} = \frac{1}{48}$$

This cannot be further simplified so this is our answer.

Let's move on to the next one.

This one is "2/15 divided by 4/5"

$$\frac{2}{15} \div \frac{4}{5}$$

Flip the divisor:

$$\frac{2}{15} \times \frac{5}{4}$$

Check to see if the numbers diagonal to each other can be further simplified:

For 2 and 4, we can divide them by 2.

For 5 and 15, we can divide them by 5.

We then get:

$$\frac{1\ \cancel{2}}{3\ \cancel{15}} \times \frac{\cancel{5}\ 1}{4\ 2}$$

$$\frac{1}{3} \times \frac{1}{2}$$

$$= \frac{1}{6}$$

If we just performed the multiplication without simplifying first:

$$\frac{2}{15} \times \frac{5}{4} = \frac{10}{60}$$

We have larger values and we need to simplify further by dividing the numerator and denominator by 10:

$$\frac{10}{60} = \frac{1}{6}$$

Decimals

Decimals are fractions where the numerator is the value expressed after the decimal point and the denominator is 10 or a power of 10.

For example:

$$0.14 = \frac{14}{100}$$

$$0.5 = \frac{5}{10}$$

Values After the Decimal Point

Units (Ones)	Decimal Point	Tenths	Hundredths	Thousandths	Ten Thousandths	Hundred Thousandths
	•					

When we look at the table, we can examine our decimal value in depth:

For example, in **1.457**

1 is the unit (ones)

4 is in the tenths place $\left(\frac{4}{10} = 0.4\right)$

5 is in the hundredths place $\left(\frac{5}{100} = 0.05\right)$

7 is in the thousandths place $\left(\frac{7}{1000} = 0.007\right)$

When we add them together: 1 + 0.4 + 0.05 + 0.007 = 1.457

Values Before the Decimal Point

Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Units (Ones)	Decimal Point
						•

For 25 378:

2 is in the ten thousands position
2 x 10 000 = 20 000

5 is in the thousands
5 x 1000 = 5000

3 is in the hundreds
3 x 100 = 300

7 is in the tens
7 x 10 = 70

8 is the unit before the decimal (also referred to as "ones"; so you can say "8" ones)

8.0

20 000 + 5000 + 300 + 70 + 8 = 25 378

Now that we reviewed the decimal values, we can apply it to our basic operations (addition, subtraction, multiplication, and division).

I will begin with an example where we multiply a decimal by 10, 100, 1000, etc.

Decimals are frequently seen in measurement conversions and drug dosage calculations.

For example, if a physician orders 0.4 g of a drug to be administered to a patient and the available tablets are 200 mg in strength, how many tablets should be given to the patient?

We know that 1 gram = 1000 milligrams so we will need to perform the conversion.

$$\frac{0.4\ g}{x\ mg} = \frac{1\ g}{1000\ mg}$$

We align the units. Grams are in the numerator and milligrams are in the denominator. We then cross multiply to solve for x:

$$(x\ mg)(1\ g) = (0.4\ g)(1000\ mg)$$

$$x = 400$$

So we see that 0.4 grams is equivalent to 400 milligrams. Since each tablet is 200 mg in strength, the patient should be given 2 tablets.

When you multiply a decimal by 10, 100, 1000, 10 000, etc., you count the number of zeros in the multiplier, and then move the decimal point of the multiplicand (the number that is being multiplied to the multiplier) to the right using the number of zeros.

For example:

The multiplier 1000 has three zeros, so we move the decimal point three places to the right in the multiplicand, 0.4:

$$0.4 \times 1000 = 0 \underset{400.}{\longrightarrow}$$
$$= 400$$

Ten has one zero, so we move the decimal point one place to the right:

$$1.52 \times 10 = 1 \underset{5.2}{\rightarrow}$$

$$= 15.2$$

One hundred has two zeros, so we move the decimal point two places to the right:

$$14.057 \times 100 = 14 \underset{05.}{\rightarrow} 7$$

$$= 1405.7$$

This is the one most frequently seen during conversions (as seen in our example). However, we will review how to multiply decimals when we are not multiplying by 10, 100, 1000, etc.

Rounding with Decimals

When you are instructed to round to the nearest decimal place (such as the nearest tenth, nearest hundredth, nearest thousandth, etc.), follow this rule:

The general rule:

If the next number that follows is less than 5, the number before it stays the same when rounding.

If the next number that follows is 5 or greater, you round up by 1.

For example: Round 4.79 to the nearest tenth.
Since our focus is on the tenths place, we look at the seven. Since we want to round to the nearest tenth, the number that follows (9) is what we use when applying the rule. Since the last number that follows is 5 or greater (in this case, it's 9), we round up by 1.

The answer is 4.8

Another example: Round 5.3 to the nearest whole number.

Again, our focus is on the nearest whole number unit so we look at the five. Since we want to round to the nearest whole number, the number that follows (3) is what we use when applying the rule. Since the last number that follows is less than 5 (in this case, it's 3), the number stays the same.

The answer is 5

One final example: Round 18.2496 to the nearest thousandth.

Since we know that the 9 is in the thousandths position and the number that follows it is 5 or greater (in this case, it's 6), we have to round up. The 9 will become a 10 which also affects the value before it (the 4).

The answer is 18.250 or 18.25. The trailing zero is usually omitted in dosages to prevent any medication errors from misreading (so 18.25 is used). **In addition, it is important to place a zero before a decimal point (a leading zero) if the dose and units are less than 1.** For example, writing 0.5 instead of .5

If 0.3 g of a drug is to be given to the patient, the nurse, pharmacist, or other healthcare provider might misread it as 3 g if it is written as .3 g on the order. This error can have severe consequences and may harm the patient. **This is why the leading zero is used for accurate reporting of doses and dosage values in the health professions.**

However, in measurements where the number of significant figures is asked, the zero can be significant. Zeros that are to the right of a decimal point are significant. If five significant figures are asked for, 18.250 has five significant figures (while 18.25 has four). Significant figures play an important role in fields such as engineering and the physical sciences.

Addition with Decimals

When adding decimals, you must first line up the decimals by writing them in a list or column form.

So if I want to add 1.25 + 0.4 + 357.2, I write it as

$$\begin{array}{r} 1.25 \\ 0.4 \\ \underline{+\ 357.2} \end{array}$$

After writing it out as a column and aligning the decimal points, perform the operation by starting from the right and moving left.

←

$$\begin{array}{r} 1.25 \\ 0.4 \\ \underline{+\ 357.2} \\ 358.85 \end{array}$$

In the final answer, place the decimal point directly under the position of where we aligned the decimal points.

Let's look at another example.

Add up the following decimals: 54, 6.4, 81.25, 0.1398

One thing that you will notice is that we now have a whole number. When there is a whole number, it is implied that the decimal comes after it. If you think you might get confused, you can write it out as 54.0 instead.

Step 1: Write it out in list/column form.

$$\begin{array}{r@{.}l} 54 & 0 \\ 6 & 4 \\ 81 & 25 \\ +\,0 & \underline{1398} \end{array}$$

(I separated them to make it easier to read and add up)

Step 2: Perform the addition operation.

Start from the right and work your way to the left. You should be able to determine where the decimal point is aligned as you work from right to left.

$$\begin{array}{r@{.}l} 54 & 0 \\ 6 & 4 \\ 81 & 25 \\ +\,0 & 1398 \\ \hline 141 & 7898 \end{array}$$

Our answer is 141.7898.

Subtraction with Decimals

When subtracting decimals, we follow similar steps to addition such as aligning the decimal points. However, there are a few new concepts we will learn that we didn't have to do when we added.

For example, subtract 0.025 from 45.

First, let's align the decimal points.

$$\begin{array}{r} 45.0 \\ \underline{-\ 0.025} \end{array}$$

For subtraction, we have to extend the zeros to make the number of digits after the decimal point equal if one of the numbers is shorter.

In our example, we want to add zeros after the decimal in 45 so that each will have three places after the decimal point. When we do that, we can begin subtracting:

$$\begin{array}{r} 45.000 \\ \underline{-\ 0.025} \end{array}$$

Using the rules of subtraction, moving from right to left, top to bottom, we get our answer: **44.975**

Let's look at a more involved example.

Evaluate the following.

$$\begin{array}{r} 25.1 \\ 12.035 \\ -\ \underline{0.4} \end{array}$$

First, we verify that the decimal points are aligned. After we do that we have to add zeros to the shorter numbers to make the number of digits after the decimal point equal.

$$\begin{array}{r} 25.100 \\ 12.035 \\ -\ \underline{0.400} \end{array}$$

We then perform the subtraction from right to left and top to bottom. We then get our answer: **12.665**

<u>Subtraction Review</u>

As we had seen in the previous section on subtraction, we subtract from right to left and top to bottom. This means that we start from the right and the top of the column and subtract downwards.

For example:

$$\begin{array}{r} 56 \\ \underline{-\ 24} \\ 32 \end{array}$$

We start at the column with the 6 and move down to subtract. We perform 6 - 4 = 2. We then line it up with the column we performed the subtraction on. Next, we move left to the five. Again, we start at the top and move down: 5 - 2= 3. We align that with the column we performed subtraction on. We get our final answer, 32.

What about if the digit at the top is less than the one at the bottom?

For example:

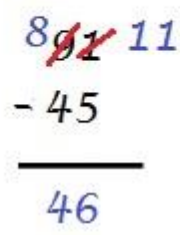
$$\begin{array}{r} 91 \\ \underline{-\ 45} \end{array}$$

1 is less than 5 here. When we do this, we “borrow” from the number next to it (in our example, it’s the tens place), and add 10 to our original number on the right. Since we borrowed from the left number (9), we take away from the tens place and it becomes 8. The 1 becomes 11.

$$\begin{array}{r} 8\,\cancel{9}\cancel{1}\ 11 \\ \underline{-\ 45} \end{array}$$

Now we can perform our subtraction:

From the right column (top to bottom): 11 - 5 = 6
Move to the left column (top to bottom): 8 - 4 = 4



We get our answer: **46**

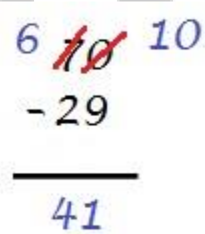
To verify your answer, add the final answer (called the **difference**) to the number above it (called the **subtrahend**). The number should equal the **minuend** (the first number we subtract from). In this case 46 + 45 = 91. So it works out correctly.

In the decimals we saw an example where we had to subtract from a “zero”:

For example:

70
<u>- 29</u>

The process is the same. We borrow from the number to the left of it. “0” becomes 10 and “7” becomes 6.



Again, let’s add the difference and the subtrahend (41 + 29). This equals the minuend, 70. We performed the subtraction correctly.

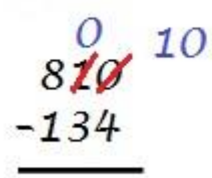
What if there are consecutive numbers we have to borrow from?

For example:

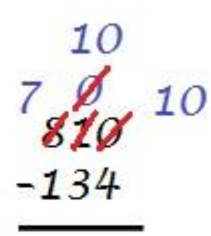
810
<u>- 134</u>

This is where it gets a little bit tricky. We borrow from the left until we can perform the subtraction at the current column without having to borrow (the digit at the top is greater than the one we are subtracting at the bottom)

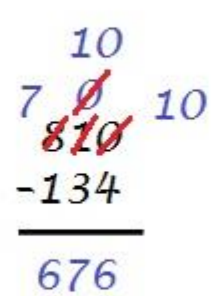
First, we start at the top right. We borrow from the left. The “0” becomes a 10 and the “1” to the left of it becomes 0.



Now the top number at the second column is 0 and the bottom number is 3. We won’t be able to subtract since the number on top is less than the number on the bottom. Again, we will borrow from the left number. We add 10 to the 0 and it becomes 10 while 8 becomes 7.



All of the numbers at the top are greater than the bottom so now we can perform the subtraction:



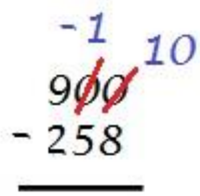
Again, we can verify through addition: 676 + 134 = 810.

Another point we will look at is when we encounter a number with consecutive zeros.

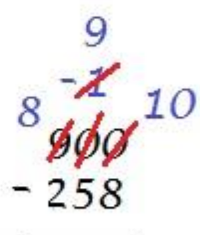
For example:

900
<u>- 258</u>

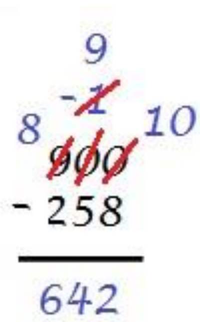
When we borrow from 0, we have to borrow again from the number next to it. In this case, the 0 to the top right borrows from the 0 next to it from the second column. Remember that we subtract 1 from the digit to the left of where we're borrowing. Mentally the 0 becomes a -1. Since that number is less than the bottom (5), we have to borrow from the 9.



Again, we borrow from the left number which adds a 10 to the one who's borrowing: -1 becomes 9 since (10 + (-1)), (simplifying to 10 - 1) = 9. Since we borrowed from the 9, it becomes 8.



Since the numbers at the top are greater than the bottom, we can perform the subtraction (right to left, top to bottom):



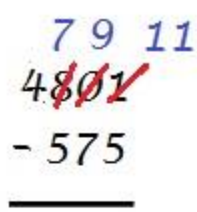
Again, let’s add to verify: 642 + 258 = 900. So we have the right answer.

There will be a stop point where you won’t have to borrow. This is case when the number on the top is greater than the one on the bottom.

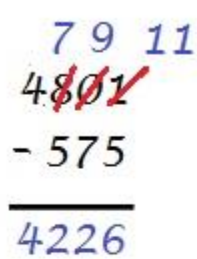
For example:

$$\begin{array}{r} 4801 \\ \underline{-\ 575} \end{array}$$

We apply all of the rules we learned so far to get:



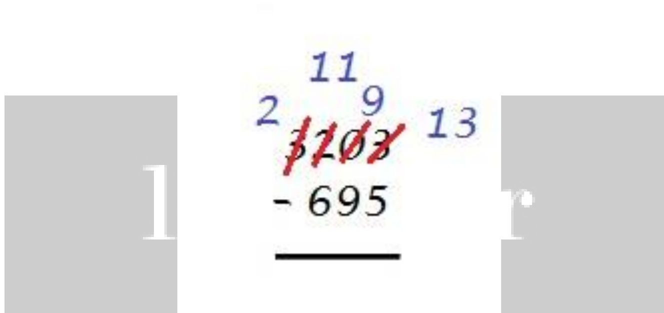
As we continue to borrow, 7 is already greater than 5. We won’t have to borrow from the number to the left of it. We can then perform our subtraction.



Again, let's add to verify: 4226 + 575 = 4801.

Let's do one final example to put everything together.

3203
<u>- 695</u>



Starting from the top right, 3 borrows from 0. 3 becomes 13 and 0 becomes -1.

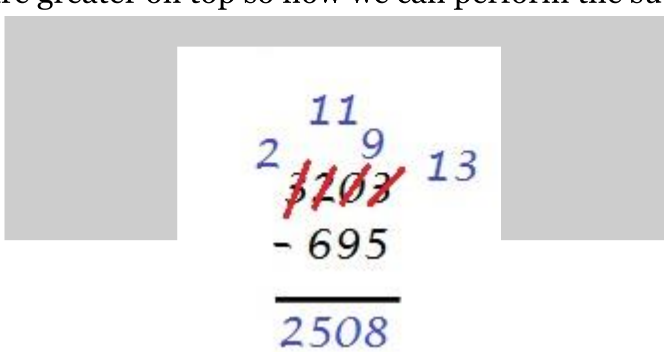
Moving left →:

Next column: the 0 (now a -1) borrows from 2 and becomes 9: (-1 + 10).

Next column: the 2 becomes a 1 since the 0 borrowed from it. The 2 borrows from the 3 and becomes 11: (1 + 10).

Next column: the 2 (now a 1) borrowed from the 3 to become 11 so now it turned it into 2.

All of the numbers are greater on top so now we can perform the subtraction:



We get our final answer, 2508. Let's verify our subtraction: 2508 + 695 = 3203.

<u>Multiplication with Decimals</u>

Multiplication with decimals is a bit simpler. You don't have to align the decimal points like what we had done during addition and subtraction. Instead we will follow a different process.

Example: Multiply 0.65 by 2.1

First, we will write it out in a list/column form.

$$\begin{array}{r} 0.65 \\ \underline{\times\ 2.1} \end{array}$$

Afterward, perform the multiplication ignoring the decimal points for now. We get our answer: 1365

To perform multiplication, you start with the lower right and multiply it with the digits above it starting from the right and moving left. We then align the first digit with our number at the bottom (1 x 5 aligns under 1) and then follow along with the digits we multiply with from the top.

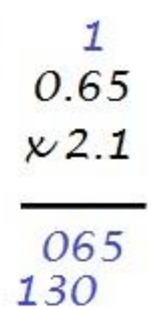
We start with 1 and multiply it by 5. Then we multiply 1 by 6. Finally, 1 by 0.

$$\begin{array}{r} 0.65 \\ \times 2.1 \\ \hline 065 \end{array}$$

Next we go to the number to the left, 2. When we multiply 2 by 5, we align with the number at the bottom (similar to when we calculated 1 x 5, it aligned below 1; this time, 2 x 5, it aligns below 2). Another way to think about it is to leave a space by aligning the next row with the second to the last digit above it.

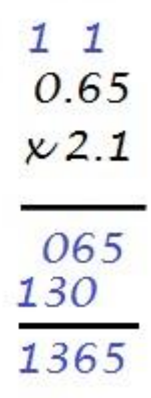
When the product has two digits (in our example 2 x 5 = 10), you put the value of the right digit under (so we align 0 under where the 2 is) and then take the left digit and place it on top of the next column we have to multiply. Again, you can align it with the second to last digit from the row above it if it's easier to remember that way.

For the next column, we multiply 2 x 6 which gives us 12. But since we carried the 1 (from the 2 x 5), we add that to the 12. The total becomes 13. We take the right digit 3 and place it next to the 0 below. We then carry the 1 and place it on top of the 0 in 0.65.



After we do that, we go to the final column. 2 x 0 = 0. Since we carried over the 1, we add that to our product: 0 + 1 = 1. We then bring it down next to the 3.

We then add everything together at the bottom using the rules of addition.



Now, count the number of decimal places after the decimal point for each number.

0.65 = 2 places
2.1 = 1 place

You then add the number of places (2 + 1 = 3 places). When we return to our answer (in our example it's 1365), we start at the end and move the decimal point to the left with whatever number of places gets added up (in our example it's 3 places to the left).

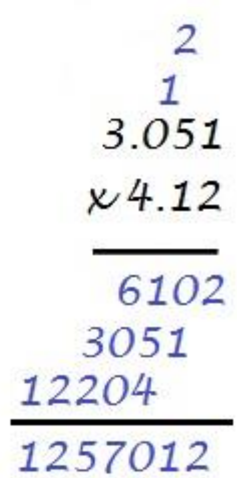
We then get our final answer **1.365**

Let's do another example.

Example: Multiply 3.051 by 4.12

$$\begin{array}{r} 3.051 \\ \underline{\times\ 4.12} \end{array}$$

Perform the multiplication ignoring the decimal points for now. We get our answer **1257012**.



Next count the number of places after the decimal point for each number.

3.051 = 3 places
4.12 = 2 places

We then add them up (3 + 2 = 5) to get 5 places. This means we will move the decimal point 5 places to the left starting from the right.

We then get our final answer: **12.57012**

If you have a whole number (e.g. 14 or 6), it counts as 0 places. So if we multiply 0.25 by 6 and ignore the decimal at first, we get 150. 6 counts as 0 places and 0.25 has two places after the decimal point. When we move to the left, we get 1.50 or simply 1.5

The methods here can also be used for integers. For example, if you want to multiply 345 by 27 (i.e. 345 x 27) or 58 by -4 (i.e. 58 x -4), you follow the same procedures above. The only thing you will skip is counting the decimal places since those are not applicable.

Division with Decimals

When dividing with decimals, follow the procedure outlined in the following example.

Evaluate: 6.25 ÷ 0.5

An easy way to remember how to set this up is to first rewrite it as

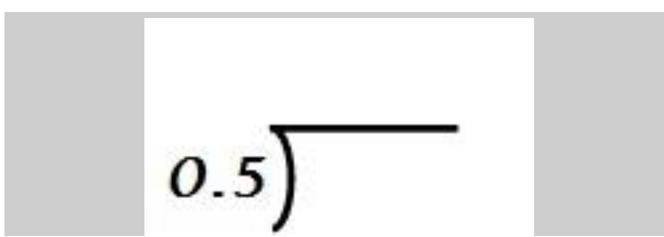
$$\frac{dividend}{divisor}$$

In our example 6.25 is the dividend and 0.5 is the divisor. Generally speaking if we say A divided by B (also as A ÷ B), A is the dividend and B is the divisor.

We rewrite it as

$$\frac{6.25}{0.5}$$

We now want to prepare this for long division. To do that, place a bar (or parenthesis, whichever you prefer) next to the divisor attached with a line segment (the vinculum) at the top. You can also think of it as shifting the original line above the divisor to the right and then putting a bar next to it. It should look like this:



You then place the dividend inside:

$$0.5\overline{)6.25}$$

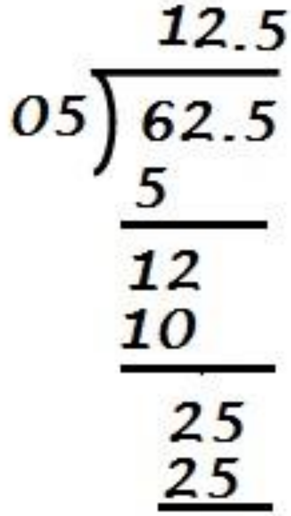
Next, move the decimal point to the right until it is at the end of the divisor. Count the number of places you moved. This is because the divisor has to be an integer. In our example, we move the decimal point one place to the right to put it in front (05.)

$$05.\overline{)6.25}$$

You then move the decimal point of the dividend to the right with the number of places you moved from the divisor. Since we moved one place to the right, we move the decimal point one place to the right in the dividend. (6.25 becomes 62.5)

$$05.\overline{)62.5}$$

Evaluate the quotient using long division:



You can omit the 0 from the divisor and just have 5 (I just left it there as an example in case you wonder where the 0 went). Also note that the decimal point aligns with the place of the decimal in the dividend.

We get our answer: **12.5**

Let's do one more example.

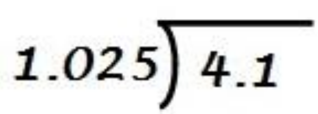
Evaluate: 4.1 ÷ 1.025

Step 1: Rewrite the problem as dividend over divisor.

$$\frac{4.1}{1.025}$$

Step 2: Organize the dividend and divisor for long division.

Again, we mentally shift the bar to the right and place a vertical bar or parenthesis next to the divisor (1.025).



Step 3: Move the decimal point to the right for the divisor to turn it into an integer. Count the number of places that you've moved and move that in the dividend.

For the divisor, we move the decimal point three places to the right to make it an integer. We then move the decimal point three places to the right for the dividend.

$$1025\overline{)4100}$$

Step 4: Perform the long division.

$$\begin{array}{r} 4 \\ 1025\overline{)4100} \\ \underline{-4100} \\ 0 \end{array}$$

The answer is 4.

<u>Review of Long Division</u>

In the last example we used long division to divide decimals. In this section, we will review the fundamentals of long division including those that have remainders. The examples in the previous section divided out evenly without any remainders.

When performing long division, the dividend is stated first and goes inside the "house" while the divisor is outside. The expression can be written in several ways.

For example, 45 divided by 9 can be written as:

45 ÷ 9

45/9

$$\frac{45}{9}$$

The **dividend** is the number we state first. It is the number we are dividing. The **divisor** is the number stated next. It tells us the number of parts we are dividing the dividend into (the number that divides). So we take our dividend 45 and divide it into groups with 9 parts each.

If I have 45 pieces of candy and divide them by giving 9 pieces per person. How many people will get 9 pieces of candy from my bag of 45 pieces?

Person A gets 9 pieces. That leaves me with 45 - 9 = 36 pieces left.

Person B gets 9 pieces. That leaves me with 36 - 9 = 27 pieces left.

Person C gets 9 pieces. That leaves me with 27 - 9 = 18 pieces left.

Person D gets 9 pieces. That leaves me with 18 - 9 = 9 pieces left.

Person E gets 9 pieces. This finishes off the bag since I will have 0 pieces left. If we divide 45 pieces of candy by giving 9 pieces to each person, 5 people will have a complete group of 9 pieces of candy.

The result that you get is called the **quotient**. In our current example, the quotient is 5.

However, what will happen if I have 30 pieces of candy and have to divide it evenly among 4 people? We will see that it won't be even like last time.

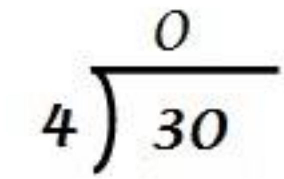
If I go around and start by giving each person 1 piece of candy, everyone will have 7 pieces of candy (7 x 4 = 28). However, there will be 2 pieces of candy left over. Two people will get another piece of candy while the other two won't. Since we want to be fair, we let everyone have 7 pieces of candy and we state that there are 2 remaining pieces.

We can use long division to evaluate this (30 ÷ 4)

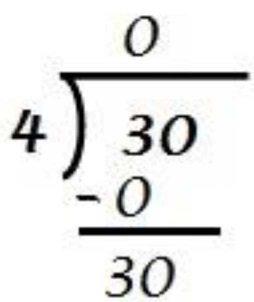
$$4\overline{)\,30}$$

You look at the divisor (in our example, 4) and the dividend. You divide the digit on the dividend by the divisor and write the result's whole number below the digit on the dividend. (You can also start with each digit to see if you can multiply a number that will be closest to the dividend without going over.) Don't worry I'll explain this in the next paragraph.

We start with the 3 and divide by 4. Since 3 is less than 4, we know that performing the division will give a number less than 1. (i.e. 3 ÷ 4 = 0.75) We use the whole number value before the decimal point and align it with the number we divided by the divisor. Since 3 was divided by 4, we put the 0 on top of the 3.



We then multiply that number by the divisor and put it under the digit it aligned with. So we multiply the 0 on top by 4 and put the digit under the 3. You then subtract that number from the digit above and then carry the digit next to it down.

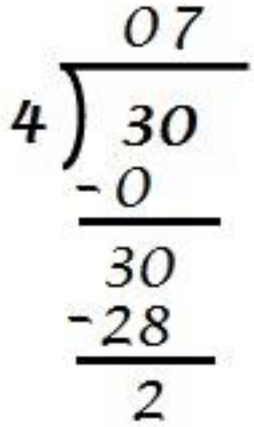


As you can see, we subtracted 0 from 3 and put the answer down. We then carry the next digit down (in this case it's 0) and put it next to our subtracted answer.

We then divide that number (30) by the divisor (4) and put the whole number value at the top. This one won't divide evenly so you put the whole number that is as close to the dividend we're dividing with and not worry about the remainder for now.

30 ÷ 4 = 7 (with 2 remaining, but we'll ignore the 2 for now)

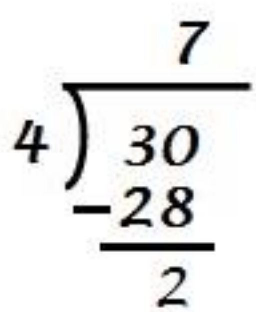
We put 7 on top and then follow the same steps by multiplying with the divisor (7 x 4):



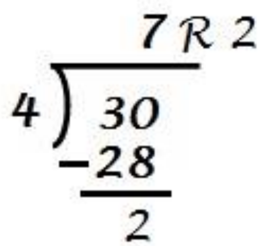
We then subtract the result of the product from the number above it (30 - 28). Since we are at the end of the dividend, we just state that **the answer is 7 with a remainder of 2**.

A shortcut is if the divisor is greater than the first digit of the dividend, move to the next digit to see if you can divide (in this case, we just skip putting the 0 in front of the 7). We see that 4 is greater than 3 so we move to the next digit and include it together with the first digit. When we do that, 4 is less than 30 so we can now perform our division. (Remember, if the divisor is greater than the dividend such as our example above when we divided 4.1 by 1.025, just mentally place a 0 until you can divide the dividend by the divisor).

Back to our example. The closest whole number we get when dividing 30 by 4 is 7. You can also check using multiplication (What number times 4 will give us a number that is close to or equal to 30? The answer is 7. 7 x 4 equals 28. If we use 8 x 4, we get 32 which is greater than our dividend.)

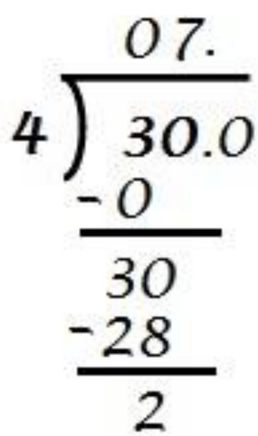


When we do this, we have a remainder of 2, just like in our example. When this happens, you can just write an R next to the quotient with the number remaining.

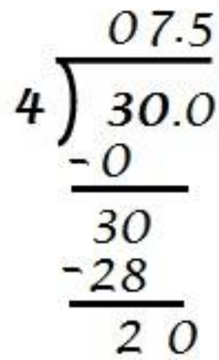


The answer is 7 remainder 2.

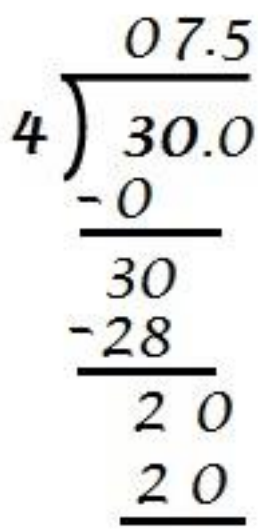
You can also use decimals to continue the division. In this case, your answer will be a decimal. To do this, we put a decimal point to the right of the dividend and add zeros. We then place the decimal point above where our integer was.



We then perform the same steps. Bring down the next digit in the dividend and divide it by the divisor. So in this example, we bring down the zero and divide 20 by 4 and put the number on top.



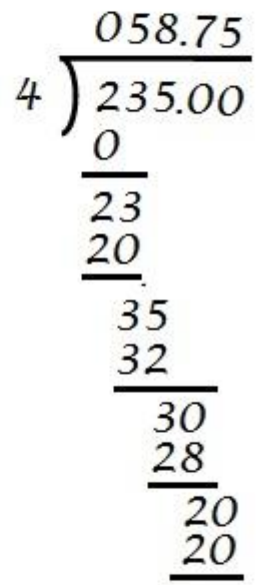
Multiply the number we placed above (in our example, it's the 5) with the divisor and finish the problem. The numbers (20 - 20) subtract to 0.



We then get our answer in decimal form: 7.5

I will do another example, but the explanation should be clear to you. Instead, I will just work out the problem.

Example: Evaluate 235 ÷ 4.



The answer is 58.75. Or if you want to use the remainder, 58 remainder 3.

To double check your answer with the remainder, multiply the integer on top with the divisor (in our example it would be 58 x 4 = 232). You then add the remainder to that number. It should give you the original dividend. 232 + 3 = 235

When the digits after the decimal point terminate without continuing infinitely, it is referred to as a **terminating decimal**. In our examples, our answers had terminating decimals.

However, when we divide certain numbers such as 1/9 or 3/7, the pattern of numbers continues infinitely:

1/9 ≈ 0.111111 …etc. or $0.\overline{1}$
(where the bar represents the repeating numbers after the decimal point)

3/11 ≈ 0.272727 …etc. or $0.\overline{27}$

these are referred to as **repeating decimals**.

Ratios and Proportions

We will encounter ratios and proportions throughout medical mathematics. For example, if we are calculating the flow rate (speed in which intravenous fluids are infused into the patient) of intravenous heparin, we can use ratios and proportions.

Ratios describe a relationship comparing two numbers. For example, if I have a bowl with 4 red pieces of candy and 5 green pieces of candy, the ratio of red pieces to green pieces is 4:5 (read as "four to five"). Whichever number is discussed first, is what comes first in the ratio. So if I asked for the ratio of green pieces to red pieces, it will be 5:4 instead.

Ratios can be written as fractions such as 4/5 or the traditional form 4:5 (using a colon).

Proportions show the relationship between two equal ratios.

$$1:2::3:6$$

In a proportion the terms which are on the outside are called the **extremes**. In our example, 1 and 6 are the extremes. The terms on the inside are called the **means**. In our example, 2 and 3 are the means.

Remember from our discussion on ratios that 1:2 = $\frac{1}{2}$. So if we turn our proportion into fraction form:

$$\frac{1}{2} = \frac{3}{6}$$

Going back to our example regarding the infusion of intravenous heparin, let's look at an example applying our knowledge of proportions.

Example: A physician gives an order for an IV infusion of D5W (5% dextrose in water) 500 mL with 20000 units of heparin at 1000 units/hour.

What is the flow rate in mL/hr?

Although we will learn formulas at a later handout, we will use the ratio and proportion method now.

We have to line up our proportion so that the units align with each other. We already know that proportions show a relationship between two equal ratios.

$$\frac{20000\ units}{500\ mL/hr} = \frac{1000\ units}{x\ mL/hr}$$

When we organize our known information using the proportion method, we know that in total 20000 units of heparin are infused with 500 mL of D5W in one hour. We now have to find an equivalent ratio (relationship) for the physician's orders. The physician's orders state that 1000 units are to be infused into the patient per hour. We have to find how many mL of D5W are to be infused with 1000 units of heparin per hour.

When we have a proportion with an unknown, we cross multiply to solve the equation. To cross multiply, just multiply the two values that are diagonal to each other.

In our example it's (20000)(x) and (1000)(500). Once we do that we should get

$$20000x = 500000$$

Isolate the unknown variable by dividing both sides by 20000:

$$\frac{\cancel{20000}x}{\cancel{20000}} = \frac{500000}{20000}$$

$$x = 25$$

The intravenous heparin flow rate is 25 mL/hr.

Another technique is that when dividing two values that have zeros at the end as above, you can remove the number of zeros from the lesser value from the greater value. For example, 20000 has four zeros. We can remove four zeros from the greater value, 500000. We then get

$$\frac{50\cancel{0000}}{2\cancel{0000}} = \frac{50}{2} = 25$$

Other examples include:

$$\frac{460}{500} = \frac{46\cancel{0}}{50\cancel{0}} = \frac{46}{50} = \frac{13}{25}$$

$$\frac{300\cancel{0}}{25\cancel{0}} = \frac{300}{25} = 12$$

So just remember to take the number of zeros from the lesser number and remove that from the greater number. As you can see the lesser or greater number can either be on the numerator or the denominator.

Later on we'll also learn dimensional analysis to calculate dosages. Dimensional analysis is used in fields such as chemistry and physics. I won't discuss that now since it might get overwhelming. The focus for this handout is to refresh one's knowledge of foundational mathematics.

<u>Percentages</u>

Percentages describe a fraction where the numerator and the denominator is 100. In terms of ratios, the second term is 100 (e.g. **X:100**).

45% in fraction form:

$$\frac{45}{100}$$

45% in ratio form:

$$45:100$$

Percents can be converted to decimals by dividing by one hundred. In this case, you take the number in the numerator and move the decimal point two spaces to the left. (Remember that the decimal point is in front of a whole number).

For example,

52% = 0.52
9% = 0.09
75.4% = 0.754

Similarly, if you are given the decimal value, you can convert it to a percent by multiplying it by 100 (i.e. moving the decimal point two places to the right).

For example,

0.78 = 78%
0.356 = 35.6%
0.085 = 8.5%

Let's look at an example where the percent is the unknown.

Example: At a store you see a coat that originally costs \$68. The store is having a sale and \$14 will be deducted from the original price. What was the percent discounted from the original price?

Usually stores will tell you how much will be discounted during sales. 30% off, 15% discount, etc. In this example we are given the amount that is already deducted (\$14). We need to find out what percent of 68 is 14. Our unknown x will be the percent.

Step 1: Use the definition of percentage to set up the unknown.

We can write it as a fraction or ratio. Writing it as a fraction will make it easier. Since we know that a fraction is a number over 100, we can set up our unknown as $x/100$. The final answer is 14 so we know that it will be to the right of the equal sign.

$$\frac{x}{100}(68) = 14$$

Step 2: Solve for *x* using algebra.

$$\frac{68x}{100} = 14$$

Multiply both sides by 100 to remove it from the left.

$$(\cancel{100})\frac{68x}{\cancel{100}} = 14(100)$$

$$68x = 1400$$

Divide both sides by 68 to isolate the x.

$$\frac{\cancel{68}x}{\cancel{68}} = \frac{1400}{68}$$

$$x = 20.59$$

The coat was discounted by 20.59%.

To double check your answer, multiply the percent by the original price (number).

Step 1: Convert the percent to decimal form.

Moving two places to the left we get 0.2059

Step 2: Multiply that by the original price (or original number).

(0.2059)(68) = 14.0012
which rounds off to 14

This verifies the price of the original discount.

Now let's look at an example where we are given the percent.

Example: In a nursing microbiology class of 110 students, a professor has decided to only give 12% of the class an A grade. How many students will receive an A?

This one is simple. You just multiply the percent by the number of students.

First, we convert the percent to decimal form. (A common mistake is multiplying by the percent without conversion; for this example, it's obvious that it's wrong since 12 x 110 = 1320 which is way off)

Continuing on, performing the conversion gives us (0.12). We then multiply that by 110.

$$(0.12)(110) = 13.2$$

Around 13 students will receive an A.

Let's do an example that won't be as straightforward.

Example: A restaurant is having a special where if you order two dishes, a 30% discount will be applied to the dish of equal or lesser value. You order a small personal size pizza that costs $5 and a pasta dish that costs $14. How much is the total for your order?

This example is simple, but can confuse you if you're not careful. The main thing to focus on is that the discount is for the dish of <u>equal or lesser value</u>. When you get your bill, you can't argue with the waiter or manager that the 30% should have been deducted from the pasta dish and not the personal pizza!

Step 1: Write your known variables.

Greater value is 14
Lesser value is 5
Discount is 30% = 0.3

Step 2: Apply the discount to the lesser value.

We know that the discount will be deducted from the dish that is equal to or costs less than the other dish.

$$(0.3)(5) = 1.5$$

$1.50 will be deducted from the dish

Now we subtract $5 - $1.50 = $3.50

Step 3: Add the cost to the non-discounted dish to find your total price.

$14 + $3.50 = $17.50

The total price of the meal is $17.50

If we had misread and deducted it from the more expensive dish:

$$(0.3)(14) = \$4.20$$

$$\$14 - \$4.20 = \$9.80$$

$$\$9.80 + \$5 = \$14.80$$

The meal would have cost less! However, that calculation would have been wrong.

Manipulating Equations

We looked at isolating the variables to solve for x (it can be any other variable such as y, z, etc.). The general rule for performing these operations when isolating the unknown variable is to **perform the operation on both sides of the equal sign**. If you're subtracting on one side, you subtract on the other. If you're dividing one side, you divide on the other. We will also see that **we need to perform the opposite operation when canceling out the other values** in the equation.

Addition example:

$$x + 12 = 34$$

Subtract the number on the side with the unknown on both sides of the equal sign. In our example, 12 is on the side with the x. Since the number 12 is positive, we subtract 12 from both sides to cancel out the 12 from the left side.

$$x + \begin{matrix} 12 \\ -12 \end{matrix} = \begin{matrix} 34 \\ -12 \end{matrix}$$

$$x = 22$$

Plug in the answer for x to verify the result: $22 + 12 = 34$

This is also the case when you perform subtraction, but the number is positive while the unknown is negative. Usually the positive number is written first (e.g. $12 - x$, $x - 30$, etc.) for stylistic purposes, but it's not mandatory. You can also write the following as $-x + 25 = 9$.

For example,

$$25 - x = 9$$

Since 25 is positive, we subtract 25 from both sides:

$$\begin{array}{c} 25 \\ -25 \end{array} - x = \begin{array}{c} 9 \\ -25 \end{array}$$

$$-x = -16$$

When the unknown variable is negative, multiply both sides by -1 to make it positive.

When we do that

$$(-1)(-x) = (-1)(-16)$$

$$x = 16$$

When we plug in our values into x: 25 - 16 = 9

Subtraction example:

$$x - 40 = 10$$

Add the number on the side with the unknown on both sides of the equal sign. In our example, we add 40 to both sides. (Another way to look at it is that -40 is negative so we have to perform addition to cancel it out)

$$x - \begin{array}{c} 40 \\ +40 \end{array} = \begin{array}{c} 10 \\ +40 \end{array}$$

$$x = 50$$

Plug in the answer for x to verify the result: 50 - 40 = 10

Multiplication example:

$$20x = 40$$

Divide both sides by the number that is multiplied together by the unknown. 20 is multiplied by the unknown variable x, so we divide both sides by 20.

$$\frac{\cancel{20}x}{\cancel{20}} = \frac{40}{20}$$

$$x = 2$$

To check if your answer is correct, plug in the answer to the original equation.

$$(20)(2) = 40$$

This works out correctly.

Division example:

$$\frac{x}{3} = 10$$

To isolate the x here, multiply both sides by 3.

$$(\cancel{3})\frac{x}{\cancel{3}} = 10(3)$$

$$x = 30$$

When you get to algebra, you can rewrite equations with fractions into integers by multiplying both sides of the equal sign by the denominator:

$$\frac{2}{5}x + 3y = 10$$

We multiply all of the terms by 5:

$$(5)\frac{2}{5}x + (5)3y = (5)10$$

$$2x + 15y = 50$$

So remember our rules: perform the opposite operation to cancel out the values when isolating the unknown variable and whatever you do on one side, you must do on the other. This is because the equation is equal. Performing an operation on one side and not on the other will make it unbalanced.

Let's finish this section by using everything we learned to isolate the unknown and solve for its value.

Example: Solve for *x*.

$$\mathbf{\frac{3x}{10} + 17 = 8}$$

Now we will need to use our rules to solve for x. First we have to get cancel out any numbers not joined with x. We see that 17 is added to the fraction with our unknown. We need to subtract both sides by 17.

$$\frac{3x}{10} = -9$$

Now that we have our fraction, we have to multiply both sides by 10 to bring down the value from the numerator.

$$3x = -90$$

Since 3 is multiplied by x, we have to divide both sides by 3:

$$\frac{\cancel{3}x}{\cancel{3}} = \frac{-90}{3}$$

$$x = -30$$

Now that we have our answer, we perform the order of operations when verifying our equation.

$$\mathbf{\frac{3x}{10} + 17 = 8}$$

$$\mathbf{\frac{3(-30)}{10} + 17 = 8}$$

Parentheses:

$$\mathbf{\frac{-90}{10} + 17 = 8}$$

Division:

$$\mathbf{-9 + 17 = 8}$$

Addition (from left to right):

8 = 8 ✓