

## Medical Mathematics

Handout 1.2
Measurements and Conversions

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In the last handout we reviewed many of the key concepts encountered in basic mathematics. We will apply many of those techniques in this handout when we learn how to convert between two different systems or find the specific value of units using the metric prefixes.

## Roman Numerals and Arabic Numbers

We will start this handout by reviewing the Roman and Arabic systems.
When working with dosage calculations, Arabic numbers are the ones commonly used. However, Roman numerals are also used in systems such as the Apothecary System (which we will discuss later on in this handout). Arabic numbers are the commonly known numbers used for counting. For example, $0,1,2$, and 3 are all Arabic numbers.

Roman numerals are expressed in letters (both uppercase and lowercase) with each letter having a specific numerical value. For example, IV and iv both equal 4.

| Roman Numeral | Arabic Number |
| :---: | :---: |
| I or i | 1 |
| V or v | 5 |
| X or x | 10 |
| L or l | 50 |
| C or c | 100 |
| D or d | 500 |
| M or m | 1000 |

Another Roman numeral that is encountered in the pharmacy setting (specifically, the Apothecary System) is ss or $\overline{\boldsymbol{s} \boldsymbol{s}}$ whose value is $\mathbf{1 / 2}$.

From the table above, we can find the values of other Roman numerals.

## Roman Numeral Rules

When working with Roman numerals, there are a few rules to keep in mind:

## - Addition

When a Roman numeral that is the same (e.g. II) or smaller (e.g. XI) is placed after the larger Roman numeral, you add them together. For the first example, II $=2(1+1=2)$ and for the second example, XI = $11(10+1=11)$.

Remember, as long as the Roman numeral that comes after each numeral is equal to or less than it, you can perform the addition. Another example, XXXV $=10+10+10+5=35$.

## - Subtraction

When a Roman numeral with a smaller value is placed before a larger value Roman numeral, you perform subtraction. In this case, you subtract the value of the smaller Roman numeral from the larger Roman numeral.

For example, $\mathrm{XL}=40(\mathrm{X}=10$ and $\mathrm{L}=50$; since X is smaller than L , we subtract it from L) Thus, $50-10=40$.

Again, please remember that as long as the Roman numeral that comes after each numeral is greater than the one before $i t$, you perform subtraction.

Another example, CM.
$\mathrm{C}=100$
$M=1000$
Since $M$ is greater than $C$ and comes after it, you subtract. Subtract C from M ( $1000-100=900$ ). Thus, $\mathrm{CM}=900$.

- Roman numerals cannot be repeated more than three times one after another (i.e. in succession)

For example, III is 3 , but IIII is not 4. In this case, you have to use the subtraction method we discussed above. Thus, IV represents 4.

VIII is 8 , but VIIII is not 9 . In this case, you have to subtract. Thus, IX is 9 .
$\mathrm{XXX}=30$, but XXXX does not equal 40. (We will look at finding the equivalence of 40 in the next bullet point.)

## - Order

So far our examples have been simple. When we work with longer Roman numerals in a sequence (such as those seen on historical dates), we have to apply the rule: Subtraction is performed first and then addition.

Let's take a look at our example, MXL. Let's look at the value for each Roman numeral first.
$M=1000$
$\mathrm{X}=10$
$\mathrm{L}=50$
Moving from left to right, we have to check for subtraction first. When we look at our example, X is less than M [or M is greater than X ] (which results in addition) so we move on for now. Next, we look at X and L . X is less than L [or L is greater than X ] so we know that we have to subtract and find the value of those Roman numerals first. When we subtract X (10) from L (50), we get 40 . Thus, $\mathrm{XL}=40$.

Now that we took care of the subtraction, we can move on to addition. $\mathrm{M}=1000$ so MXL $=1000+40=1040$.

The value of the Roman numeral MXL equals 1040.
Again, the order is important similar to when we performed the order of operations. If we had moved from left to right using addition: MXL: MX $[1000+10=1010]$ and then add that to $\mathrm{L}:[1010+50=1060]$, our answer would have been completely wrong!

Now that you have had practice with Roman numerals and their equivalencies in the Arabic system, let's do a more complicated example that ties everything together.

## Example $1.2 a$

Students in a pathology class are watching a video clip on influenza. After the video finishes, the copyright date of the video clip is written as MCMXCIX. What is the year in Arabic numbers?

Step 1: Look at the Roman numeral from left to right and check to see where you can perform subtraction.

As you do this, take a look at a pair of Roman numerals. If the Roman numeral that comes after it is greater, perform the subtraction.
$\mathbf{M C}=\mathbf{C}$ is less than $\mathbf{M}$ so that will be addition. We skip that one for now.
$\mathbf{C M}=M$ is greater than C so this will be subtraction. We perform the subtraction. Since we already saw in an example that $C M=900$, we can keep that in mind and continue forward.
$\mathbf{X C}=\mathrm{C}$ is greater than X so this will be subtraction. $\mathrm{X}=10$ and $\mathrm{C}=100.100-10=90$. Thus, $\mathrm{XC}=90$.

At this point, we only have one pair left (IX) as well as the M from the beginning.
$\mathbf{I X}=\mathrm{X}$ is greater than I so this will be subtraction. $\mathrm{I}=1$ and $\mathrm{X}=10.10-1=9$. Thus, IX $=9$.
*Remember that after you perform the subtraction, note the equivalent number of the pair of numerals right away. Otherwise, you might get confused once you start performing the addition. You won't be able to use those letters separately again when you perform the addition step. An easy way to organize this is to place them in parentheses during the addition step. We will see this in action.

Step 2: Moving from left to right, perform the addition.
$M=1000$
CM $=900$
$\mathrm{XC}=90$
IX $=9$
The addition will look like this:
$\mathrm{M}+(\mathrm{CM})+(\mathrm{XC})+(\mathrm{IX})=$
$1000+900+90+9=1999$
In Arabic numbers, the year MCMXCIX is equivalent to 1999.

To summarize everything:
If the numerals that come after each individual numeral are less than or equal to it, you just perform addition:
(moving left to the right, the numerals move from larger to smaller and then are equal towards the end)
XVIII $=10+5+1+1+1=18$
If you move left to right and encounter a smaller Roman numeral before a larger Roman numeral, perform the subtraction to get it out of the way. Let's do a final example on this:

## MCDXCII

We encounter the first one at CD. At MC, it goes from larger to smaller so that will be addition. For CD we encounter a "bump" where it goes from smaller to larger. We perform the subtraction to get it out of the way: $C D=500-100=400$.

We encounter another "bump" where it goes from smaller to larger at XC. XC as we had learned equals 90 .

The final Roman numerals are the same II. So II $=2$.
Applying our techniques from our previous example:
MCDXCII $=\mathrm{M}+(\mathrm{CD})+(\mathrm{XC})+\mathrm{I}+\mathrm{I}$
$=1000+400+90+2=1492$

## Temperature

In the health services field, you will encounter two scales used as units of measurement for temperature: Fahrenheit and Celsius (Centigrade). Centigrade was the name used for Celsius in the past and is not used as frequently today.

Fahrenheit is the scale used in the United States and its territories.

- The freezing point of water is $32^{\circ} \mathrm{F}$.
- The boiling point of water is $212^{\circ} \mathrm{F}$.

Celsius is the scale used in most of the countries around the world where the International System of Units (SI) is adopted.

- The freezing point of water is $0^{\circ} \mathrm{C}$.
- The boiling point of water is $100^{\circ} \mathrm{C}$.

To convert between the two systems you use

$$
\begin{aligned}
{ }^{\circ} \mathrm{F} & =\frac{9}{5} \mathrm{C}+32 \\
{ }^{\circ} \mathrm{C} & =\frac{5}{9}[F-32]
\end{aligned}
$$

You can always use algebra to manipulate the variables in case you have a hard time memorizing both formulas.

Both of these are derived from the equation:

$$
5 F=9 C+160
$$

(At the end of this section, I will show you how we get this equation using algebra and the information from the freezing and boiling points of water)

Fahrenheit is the simpler one so we will work on that first.

## Fahrenheit:

$$
5 \mathrm{~F}=9 \mathrm{C}+160
$$

Divide both sides by 5 so that F is isolated on the left:

$$
\begin{aligned}
\frac{5}{5} F & =\frac{(9 C+160)}{5} \\
F & =\frac{9}{5} C+\frac{160}{5} \\
F & =\frac{9}{5} C+32
\end{aligned}
$$

## Celsius:

Celsius is a little more involved, but it just requires algebra: performing the opposite operations to isolate the variable and factoring out

$$
5 F=9 C+160
$$

Looking at the equation we want to isolate the C on the right side. First we will need to subtract 160 from both sides:

$$
\begin{gathered}
5 F \\
-\mathbf{1 6 0} \\
5 F-9 C+\begin{array}{c}
160 \\
-\mathbf{1 6 0}
\end{array} \\
5 F-160=9 C
\end{gathered}
$$

Next we divide both sides by 9 so that we can isolate the C :

$$
\frac{(5 F-160)}{9}=C
$$

Finally, factor out the numerator on the left side of the equation to simplify further:

$$
\begin{aligned}
& \frac{5(F-32)}{9}=C \\
& C=\frac{\mathbf{5}}{9}(F-32)
\end{aligned}
$$

As you can see, you can find both of the formulas for Fahrenheit and Celsius from the equation above ( $5 \mathrm{~F}=9 \mathrm{C}+160$ ). It's usually easier to remember this one since you just work with integers and apply algebraic concepts instead of having to memorize the fractional components (which can get mixed up) from the beginning.

Let's work on an example together.

## Example 1.2b

A patient is experiencing a fever. The nurse takes the patient's temperature and the thermometer reads $101.3^{\circ} \mathrm{F}$. What is this temperature in Celsius?

Using the standard formula:

$$
\begin{gathered}
{ }^{\circ} \mathbf{C}=\frac{5}{9}[\boldsymbol{F}-\mathbf{3 2}] \\
{ }^{\circ} \mathrm{C}=\frac{5}{9}[101.3-32] \\
{ }^{\circ} \mathrm{C}=\frac{5}{9}[69.3] \\
{ }^{\circ} \mathrm{C}=\frac{346.5}{9}
\end{gathered}
$$

$$
{ }^{\circ} \mathrm{C}=38.5
$$

The patient's temperature reads $38.5^{\circ} \mathrm{C}$.

## Example $1.2 b$

A patient looking to improve his physical health decides to do some walking as exercise since it was a warm day outside. The weather forecast stated that the temperature was $21^{\circ} \mathrm{C}$. What is the temperature in Fahrenheit?

Using the standard formula:

$$
\begin{aligned}
& { }^{\circ} \mathbf{F}=\frac{9}{5} \mathbf{C}+32 \\
& { }^{\circ} \mathrm{F}=\frac{9}{5}(21)+32 \\
& { }^{\circ} \mathrm{F}=\frac{189}{5}+32 \\
& { }^{\circ} \mathrm{F}=37.8+32 \\
& { }^{\circ} \mathrm{F}=69.8\left(70^{\circ}\right)
\end{aligned}
$$

The temperature outside was around $70^{\circ} \mathrm{F}$.

In some cases, you may use your knowledge of temperature from early science classes to find an equation and just substitute a value into $x$ to perform the conversion.

If you are familiar with algebra, we will be using a linear equation to express the relationship between Fahrenheit and Celsius.

Earlier we noted that the freezing point of water is $0^{\circ} \mathrm{C}$ and $32^{\circ} \mathrm{F}$. We also noted that the boiling point of water is $100^{\circ} \mathrm{C}$ and $212^{\circ} \mathrm{F}$.

We can express the freezing point and boiling point as ordered pairs (where $x$ is the Celsius values and $y$ is the Fahrenheit values):

## Freezing point:

$$
\left(x_{1}, y_{1}\right)=(0,32)
$$

## Boiling point:

$$
\left(x_{2}, y_{2}\right)=(100,212)
$$

Since we are trying to find a linear equation, we can find the slope using the two points above. Using the formula for slope, $m$ :

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
m & =\frac{212-32}{100-0} \\
& =\frac{180}{100} \\
& =\frac{18}{10}=\frac{9}{5}
\end{aligned}
$$

So our slope, $m=9 / 5$.
Now we have to find the $y$-intercept. We can do this by using slope intercept form:

$$
y=m x+b
$$

We already found our slope, $m$. For $x$ and $y$ we can use any of the points above (either freezing point or boiling point; both will give you the same solution).

Let's start with freezing point since the numbers are simpler to work with:

$$
\begin{gathered}
\boldsymbol{y}=\boldsymbol{m} \boldsymbol{x}+\boldsymbol{b} \\
32=(9 / 5)(0)+\boldsymbol{b} \\
\mathbf{b}=32
\end{gathered}
$$

Now let's also use the boiling point to show that the solution will be the same:

$$
\begin{gathered}
\boldsymbol{y}=\boldsymbol{m} \boldsymbol{x}+\boldsymbol{b} \\
212=(9 / 5)(100)+\boldsymbol{b} \\
212=\frac{900}{5}+b
\end{gathered}
$$

Multiply both sides (left and right of equal sign) by 5 to simplify the fraction:

$$
212(5)=\frac{900}{5}(5)+b(5)
$$

$$
1060=900+5 b
$$

Subtract 900 from both sides:

$$
160=5 \mathrm{~b}
$$

Divide both sides by 5 :

$$
\frac{160}{5}=\frac{5 b}{5}
$$

$$
b=32
$$

Now that we have all of the elements we can write our linear equation:

$$
y=\frac{9}{5} x+32
$$

Now we can note that the temperature in Celsius $=x$ and Fahrenheit $=y$. We saw this above when $x_{1}$ and $x_{2}$ represented Celsius while $y_{1}$ and $y_{2}$ represented Fahrenheit when we put down the ordered pairs for the freezing and boiling points. The final linear equation above matches with our original formula for Fahrenheit.

If you want to get Celsius from the Fahrenheit equation above, just isolate the $x$ to one side:

$$
y=\frac{9}{5} x+32
$$

Subtract 32 from both sides:

$$
y-32=\frac{9}{5} x
$$

Divide both sides by 9/5:

$$
\frac{y-32}{9 / 5}=\frac{\frac{9}{5}}{\frac{9}{9}} x
$$

Divide by multiplying by the reciprocal of the second term:

$$
\frac{y-32}{1} \div \frac{9}{5}
$$

$$
\begin{aligned}
& =\frac{y-32}{1} \times \frac{5}{9} \\
& =\frac{5(y-32)}{9} \\
& x=\frac{5}{9}(y-32)
\end{aligned}
$$

This matches our equation for Celsius when we substitute C for $x$ and F for $y$.
To see where the equation $5 \mathrm{~F}=9 \mathrm{C}+160$ came from, just multiply both sides of the Fahrenheit equation by 5 to simplify the fraction form:

$$
y(5)=(5) \frac{9}{5} x+32(5)
$$

$$
5 y=9 x+160
$$

Let $y=$ Fahrenheit (F) and let $x=\operatorname{Celsius}(\mathrm{C})$ :

$$
5 \mathrm{~F}=9 \mathrm{C}+160
$$

## Metric System

Earlier in this handout we discussed the International System of Units (SI) which is also known as the metric system. The metric system has been adopted by most countries around the world as the standard measurement system. Although the United States has not formally adopted the use of the metric system, it is widely used in the sciences and the healthcare fields.

The gram is the standard unit of weight for measuring solids.
The liter (litre) is the standard unit of volume for measuring liquids.
The meter (metre) is the standard unit of length for measuring values such as height or distance.

When you go for a physical checkup, the healthcare professional will take down your information using this system. For example, the professional will measure your height using meters (or feet) such as 160 cm ( 5 feet 2 inches) or your weight such as 50 kilograms (110 pounds).

We will take a look at the units for weight, volume, and length and their equivalencies. From there we will learn how to convert between these metric units.

## Weight

$1 \operatorname{gram}(\mathrm{~g})=$

- $1 \times 10^{6} \mathrm{mcg}$
- $1000 \mathrm{mg}\left(1 \times 10^{3} \mathrm{mg}\right)$
- $0.001 \mathrm{~kg}\left(1 \times 10^{-3} \mathrm{~kg}\right)$

1 microgram $(\operatorname{mcg})=$

- $0.001 \mathrm{mg}\left(1 \times 10^{-3} \mathrm{mg}\right)$
- $1 \times 10^{-6} \mathrm{~g}$
- $1 \times 10^{-9} \mathrm{~kg}$

1 milligram $(\mathrm{mg})=$

- $1000 \mathrm{mcg}\left(1 \times 10^{3} \mathrm{mcg}\right)$
- $0.001 \mathrm{~g}\left(1 \times 10^{-3} \mathrm{~g}\right)$
- $1 \times 10^{6} \mathrm{~kg}$

1 kilogram $(\mathrm{kg})=$

- $1 \times 10^{9} \mathrm{mcg}$
- $1 \times 10^{6} \mathrm{mg}$
- $1000 \mathrm{~g}\left(1 \times 10^{3} \mathrm{~g}\right)$


## Volume

## Length

1 liter (litre) [1 or L] =

- $1000 \mathrm{ml}($ or mL$)\left[1 \times 10^{3} \mathrm{ml}\right]$
- 1000 cc (cubic centimeters) [ $1 \times 10^{3}$ cc]

1 milliliter $(\mathrm{ml}$ or mL$)=1$ cubic centimeter
(cc) $=$

- $0.001 \mathrm{~L}\left(1 \times 10^{-3} \mathrm{~L}\right)$

1 meter $($ metre $)=$

- 1000 mm
- 100 cm

1 millimeter $(\mathrm{mm})=$

- 0.1 cm
- 0.001 m

1 centimeter $(\mathrm{cm})=$

- 10 mm
- 0.01 m

It might seem daunting having to memorize all of this information, but you can use the metric prefixes to help you determine the value of the unit. As we had discussed earlier on in this section, the base (or standard) unit from which we determine the value of the units attached to prefixes are gram for weight, liter for volume, and meter for length.

In the table above we looked at units such as milliliter, kilogram, and centimeter. When we attach the metric prefixes to the base unit, we get the value

First, let's review the SI/metric prefixes which we have covered so far:

## micro-

one millionth of a unit ( $1 \times 10^{-6}$ of a unit)

## milli-

one thousandth of a unit ( $1 \times 10^{-3}$ of a unit)

## centi-

one hundredth of a unit ( $1 \times 10^{-2}$ of a unit)

## kilo-

one thousand units ( 1000 )
So if we look at a kilogram, we know that it is one thousand grams. You can then extend it to kilometer when we want to measure distance. Although I didn't put it in the table above, we can determine that kilo + meter $=$ kilometer $=$ one thousand meters.

For milliliter, the base unit is the liter. So when we attach milli to it, we know that the value of the unit is one thousandth of a liter ( $1 \times 10^{-3} \mathrm{~L}$ or 0.001 L ).

There are three other metric prefixes which we haven't covered in previous examples:

## deci-

one tenth of a unit (o.1)

## deka-

ten units (10)

## hecto-

one hundred units (100)
Examples:
decimeter $=$ one tenth of a meter $=0.1$ meters
dekaliter $=$ ten liters $=10$ liters
hectogram $=$ one hundred grams $=100$ grams

As you can see, knowing the prefix can help you identify the value of the unit. All you need to know are the standard units of measurement (meter, liter, gram) and attach the prefix replacing the word "unit" in the definition with one of the three standard units we discussed.

In regards to healthcare, the United States also uses pounds (lb.) for weight and feet and inches for height.

To convert between the metric system and the system used in the United States:

$$
\begin{gathered}
1 \mathrm{~kg}=2.2 \mathrm{lb} \\
1 \mathrm{lb}=454 \mathrm{~g} \\
\mathbf{1} \text { foot }=12 \text { inches }=\mathbf{0 . 3 0} \text { meters }
\end{gathered}
$$

$$
1 \text { in }=0.0254 \text { meters }=2.54 \mathrm{~cm}
$$

When given a height such as the following:
(stated as) 5 foot 3 or 5 feet three inches can be written as $\mathbf{5}^{\prime} \mathbf{3}^{\prime \prime}$ where a prime symbol (') comes after the value in feet and the double prime symbol (") come after the value in inches.

To convert height measurements in this form to the metric system:
Step 1: Take the number of feet and multiply that number by 12 to convert the feet to inches.

In our example, the patient's height is $5^{\prime} 3^{\prime \prime}$. We multiply 5 by $12(5 \times 12)$ and we get 60 inches.

Step 2: Take the number of inches you calculated from the feet conversion and add that to the number of inches.

5 feet $=60$ inches
We have an additional 3 inches to add.
$5^{\prime} 3^{\prime \prime}=60+3=63$ inches
Step 3: Take the total number of inches and multiply it by 2.54 to convert it to centimeters.

Since height is usually measured in centimeters using the metric system, we multiply that number by 2.54:
$63 \times 2.54=160.02$

## So a height of $5^{\prime} 3^{\prime \prime}$ is equivalent to 160 cm .

## Household System

The household system is often used at a home setting. When patients are taking their medication at home they may refer to it as "one tablespoon" of cough syrup. This system is not used in dosage calculations in the healthcare field due to its inaccurate measurements.

Although this may seem convenient, it can become problematic due to the variations of size with the measuring instrument (thus leading to inaccurate measurements). For example, the directions on a medication bottle state that a dosage for a 7 year old child is 15 ml every 4 hours and the parent decides uses a soup spoon instead thinking it will equal 1 tablespoon (tbsp.).

We learned that 1 teaspoon ( tsp ) is equivalent to around 5 ml and 1 tbsp . is equivalent to around 15 ml . The soup spoon might hold less than expected (e.g. 7 ml ) and the child will only get half of the dosage. This is why it is best to use the calibrated instruments provided with the medication.

The metric system is what we will mainly use as we do further examples. However, I will put the metric equivalencies of the household measurements here for your reference.

| Household Measure Unit | Metric Equivalencies |
| :---: | :---: |
| 1 teaspoon | 5 ml (approximately 4.929 ml ) |
| 1 tablespoon (= 3 teaspoons) | 15 ml (approx. 14.787 ml ) |
| 1 fluid ounce (= 2 tablespoons) | 30 ml (approx. 29.5735 ml ) |
| 1 cup (= 8 fluid ounces) | 240 ml (approx. 236.588 ml ) |
| 1 pint (= 2 cups) | 480 ml (approx. 473.176 ml ) |
| 1 quart (= 2 pints) | 960 ml (approx. 946.353 ml ) |
| 1 gallon (= 4 quarts) | 3785 ml (approx. 3785.41 ml ) |

## The Apothecary System

The apothecary system has been used in the past and is not as frequently used in modern times. Although the metric system is predominantly used today, the information here can serve as a reference when you do encounter it in your field. There will be two units you should know in this system: the grain and the dram.

The standard unit of measurement for weight is the grain (abbreviated as gr). $\mathbf{1}$ grain is equivalent to 60 mg . (In approximate terms, 1 grain $=64.8 \mathrm{mg}$ )

A dram (abbreviated as dr) is also a unit of weight in this system. 1 dram is equivalent to 60 grains.

A well-known example involves Bayer aspirin. Bayer aspirin provides equivalencies in grains. For example, a tablet with 81 mg of aspirin is equivalent to 1.25 grains of aspirin (in that tablet). We will do an example together in the final section on converting between milligrams and grains.

## Avoirdupois System

The Avoirdupois system was a system of weights used in the United Kingdom.
In the United States it is often seen when shopping for groceries or items weighed using a scale. For example, fresh bread rolls may weigh 12 oz . Cherries may be advertised as $\$ 1.50$ per pound. A gallon of iced tea will have a weight of 128 fl . oz.

Here are equivalencies that are useful to reference:

$$
16 \text { ounces }(\mathrm{oz})=1 \text { pound (lb) }
$$

1 pound (lb) = 454 grams (g) [approx. 453.59 g ]
1 pound (lb) = 7000 grains (gr)
1 pound (lb) = 256 drams (dr)

## Units of Measure for Medications

There will be special types of medications which are measured in unique units.
International units (abbreviated IU) - This unit of measurement (standardized through international agreements) is based on the quantity (amount) of a biologically active substance needed to produce a specific effect. Drugs measured in international units include insulin and certain vitamins (e.g. A, D, and E).

Units- This unit of measurement is based on the quantity (amount) of a biologically active substance needed to produce a specific effect. Drugs measured in units include antibiotics and heparin as well as insulin.

- Milliunits- from the prefixes we learned above, we can note that a milliunit is one thousandth of a unit; oxytocin (brand name Pitocin is measured in milliunits)

Milliequivalents (abbreviated meq)- This unit of measurement is based on the amount/concentration of compounds found in biological fluids or IV solutions. Examples of these compounds are electrolytes such as potassium, magnesium, sodium, chloride, and calcium.

The good thing about these special units is that you won't have to perform calculations to convert them into other units in different systems (as we had seen from the metric, household, and avoirdupois systems). For example, when we calculate the flow rate for intravenous heparin, we won't have to convert the units into another system.

Since there won't be conversions required for these types of problems, we won't cover them in this handout (which focuses on measurement conversions). However, we will touch upon these problems regarding special units of medication once we get to the more involved dosage calculation problems.

For now, the most important point to know is that these medications have their own special types of measurement.

## Conversion Problems

Now that we took a look at the standard units of measure in the metric system as well as the metric prefixes, we can learn how to perform conversions between different units or different systems.

## Example 1.2c

## A physician abroad examines a file of an American pediatric patient under her current care and notes that she weighs 87 lbs . How much does the patient weigh in kilograms?

Earlier we noted that $1 \mathrm{~kg}=2.2 \mathrm{lb}$. We can use the proportion method that we learned from the previous handout.

Remember that similar units should align with each other:

$$
\frac{1 \mathrm{~kg}}{2.2 \mathrm{lb}}=\frac{x \mathrm{~kg}}{87 \mathrm{lb}}
$$

If you recall from when we discussed proportions, this example will be written as

$$
1: 2.2:: x: 87
$$

The means (the inside terms) are 2.2 and x .
The extremes (the outside terms) are 1 and 87 .
We then cross multiply and disregard the units for now:

$$
\begin{aligned}
(1 \mathrm{~kg})(87 \mathrm{lb}) & =(x \mathrm{~kg})(2.2 \mathrm{lb}) \\
87 & =2.2 x
\end{aligned}
$$

As you can see, when we cross multiply, the means are multiplied together and the extremes are multiplied together.

Divide both sides by 2.2 to isolate the $x$ to one side:

$$
\frac{87}{2.2}=\frac{2.2 x}{2.2}
$$

$$
x=39.5 \mathrm{~kg}
$$

The child weighs 39.5 kg .

## Example $1.2 d$

A patient experiencing a moderate headache visits his pharmacist to ask for medication counseling. After the session, the pharmacist gives the patient a pack of over the counter (OTC) aspirin tablets with each tablet containing 5 grains of aspirin.

## How many milligrams of aspirin are in each tablet?

(For this example, I will use the 64.8 mg instead of the 60 mg to be more accurate)

## Step 1: Identify the known information.

If you are familiar with my mathematics handouts, labeling your known information makes it easier to organize your thought process.

In our example, we note that each tablet contains 5 grains of aspirin. Another point of information that will be beneficial to us is $\mathbf{1}$ grain $=\mathbf{6 4 . 8} \mathbf{~ m g}$ (or 60 mg if you decide to use that instead). Our unknown, $x$, will be the milligram equivalent of 5 grains.

## Step 2: Set up your problem using proportions and solve.

Similar to Example 1.2c, we can set up the problem as a proportion:

$$
1: 64.8:: 5: x
$$

As fractions it should look like this:

$$
\frac{1 \mathrm{grain}}{64.8 \mathrm{mg}}=\frac{5 \mathrm{grains}}{x \mathrm{mg}}
$$

The means ( 64.8 and 5 ) should multiply together and the extremes ( 1 and $x$ ) should multiply together.

$$
\begin{gathered}
(64.8 \mathrm{mg})(5 \mathrm{gr})=(1 \mathrm{gr})(x \mathrm{mg}) \\
x=324 \mathrm{mg}
\end{gathered}
$$

## Each tablet contains 324 mg of aspirin.

## Example 1.2e

## Perform the following conversions.

a. A pharmacist has a bottle of medication with a concentration of $125 \mathrm{mg} / 5 \mathrm{ml}$. What amount in milliliters is needed to give a dose of 450 mg ?
b. A physician advises a patient to drink 2.5 liters of water per day and reduce the intake of soft drinks. How many cups of water does the patient need to drink per day?
c. A bottle contains 237 milliliters of cough syrup. How many teaspoons of cough syrup are in each bottle?
d. A person is shopping for vegetables and buys 3 large potatoes. After weighing the bag of potatoes on the scale, the weight was 3.6 lb . How many grams does the bag of potatoes weigh?

## Part a.

## Step 1: Identify/gather the known information.

The original container has a concentration of $125 \mathrm{mg} / 5 \mathrm{ml}$ of a drug.
We need to find an equivalent dose containing 450 mg of the drug. Our unknown is the volume in milliliters. The final adjusted dose will be $450 \mathrm{mg} / x \mathrm{ml}$ of the drug.

## Step 2: Perform the conversion.

The easiest way to do this is through the proportion method.

$$
\begin{aligned}
\frac{125 \mathrm{mg}}{5 \mathrm{ml}} & =\frac{450 \mathrm{mg}}{x \mathrm{ml}} \\
(125 \mathrm{mg})(x \mathrm{ml}) & =(450 \mathrm{mg})(5 \mathrm{ml}) \\
125 x & =2250 \\
\frac{125 x}{125} & =\frac{2250}{125} \\
x & =18 \mathrm{ml}
\end{aligned}
$$

18 ml are required to give a 450 mg dose for the drug.

## Part b.

## Step 1: Identify/gather the known information.

The physician has advised the patient to drink 2.5 liters of water per day.
The unknown is the number of cups equivalent to 2.5 liters. From the section on the household system, we learned that 1 cup $=240 \mathrm{ml}$ (approx. 236.59 ml ).

From the metric conversions, we learned that 1 liter $=1000$ milliliters.
With this information, we can now perform the conversion.

## Step 2: Perform the conversion.

First let's start with the conversion from liters to milliliters:

$$
\frac{1 \mathrm{~L}}{1000 \mathrm{ml}}=\frac{2.5 \mathrm{~L}}{x \mathrm{ml}}
$$

Perform the cross multiplication:

$$
\begin{aligned}
(1 \mathrm{~L})(x \mathrm{ml}) & =(2.5 \mathrm{~L})(1000 \mathrm{ml}) \\
x & =2500 \mathrm{ml}
\end{aligned}
$$

Now that we converted the units into milliliters, we can do our final conversion into cups.
(Using 240 ml )

$$
\begin{aligned}
& \frac{1 \text { cup }}{240 \mathrm{ml}}=\frac{x \operatorname{cup}}{2500 \mathrm{ml}} \\
&(1 \mathrm{c})(2500 \mathrm{ml})=(x \mathrm{c})(240 \mathrm{ml}) \\
& 240 x=2500 \\
& \frac{z 40 x}{240}=\frac{2500}{240} \\
& \boldsymbol{x}=10.42 \mathrm{cups}
\end{aligned}
$$

(Using 236.59 ml )

$$
\frac{1 \text { cup }}{236.59 \mathrm{ml}}=\frac{x \text { cup }}{2500 \mathrm{ml}}
$$

$$
(1 \mathrm{c})(2500 \mathrm{ml})=(x \mathrm{c})(236.59 \mathrm{ml})
$$

$$
236.59 x=2500
$$

$$
\frac{236.59 x}{236.59}=\frac{2500}{236.59}
$$

$$
x=10.57 \text { cups }
$$

The patient has to drink around 10 and a half cups of water per day.

## Part c.

## Step 1: Identify/gather the known information.

The bottle contains 237 ml of cough syrup. We have to find its equivalency in teaspoons.
We learned earlier that $1 \mathrm{tsp}=5 \mathrm{ml}$ (or 4.929 ml to be more accurate)

## Step 2: Perform the conversion.

$$
\begin{aligned}
\frac{1 t s p}{5 \mathrm{ml}} & =\frac{x t s p}{237 \mathrm{ml}} \\
(5 \mathrm{ml})(x \mathrm{tsp}) & =(1 \mathrm{tsp})(237 \mathrm{ml}) \\
\frac{5 x}{5} & =\frac{237}{5} \\
x & =47.4 \mathrm{tsp}
\end{aligned}
$$

The bottle has around 47 teaspoons of cough syrup.
If you decide to use the 4.929 ml instead:

$$
\frac{1 t s p}{4.929 m l}=\frac{x t s p}{237 m l}
$$

$$
(4.929 \mathrm{ml})(x \mathrm{tsp})=(1 \mathrm{tsp})(237 \mathrm{ml})
$$

$$
\frac{4.929 x}{4.929}=\frac{237}{4.929}
$$

$$
x=48.1 \mathrm{tsp}
$$

The measurements should be close to each other in either case.

Part d.

## Step 1: Identify/gather the known information.

Weight of the bag of potatoes: 3.6 lb
Conversion: $1 \mathrm{lb}=454 \mathrm{~g}$

Step 2: Perform the conversion.

$$
\begin{gathered}
\frac{1 \mathrm{lb}}{454 g}=\frac{3.6 \mathrm{lb}}{x g} \\
(1 \mathrm{lb})(x \mathrm{~g})=(454 \mathrm{~g})(3.6 \mathrm{lb}) \\
x=1634.4 \mathrm{~g}
\end{gathered}
$$

## The weight of the bag of potatoes is around 1634 grams.

## Example $1.2 f$

A physician gives an order for a single dose of diphenhydramine of $5 \mathrm{mg} / \mathrm{kg}$ body weight to a pediatric patient. If the child weighs 55 lb , how many milligrams of diphenhydramine is going to be administered to him?

## Step 1: Gather the known information.

When we look at the problem, our unknown is the amount in milligrams of the dose of diphenhydramine.

Our known information is the following:
Dose: $5 \mathrm{mg} / \mathrm{kg}$ body weight

## Child's weight: 55 lb

Looking at our information, our problem is that the child's weight is in pounds. Since the dose by body weight is $5 \mathrm{mg} / \mathrm{kg}$, we won't be able to set up a proportion until the child's weight is converted into kilograms.

## Step 2: Perform any necessary conversions.

First we will need to convert 55 lb to kg . We learned that $1 \mathrm{~kg}=2.2 \mathrm{lb}$. From this we can set up a proportion, cross multiply, and solve for $x$.

$$
\frac{1 \mathrm{~kg}}{2.2 \mathrm{lb}}=\frac{x \mathrm{~kg}}{55 \mathrm{lb}}
$$

After cross multiplying, we get the following:

$$
\begin{gathered}
(2.2 \mathrm{lb})(x \mathrm{~kg})=(1 \mathrm{~kg})(55 \mathrm{lb}) \\
2.2 x=55 \\
\frac{z .2 x}{z .2}=\frac{55}{2.2} \\
x=25 \mathrm{~kg}
\end{gathered}
$$

The child weighs 25 kilograms.
Step 3: Perform the calculations using proportions.

$$
\begin{gathered}
\frac{5 \mathrm{mg}}{1 \mathrm{~kg}}=\frac{x \mathrm{mg}}{25 \mathrm{~kg}} \\
(5 \mathrm{mg})(25 \mathrm{~kg})=(1 \mathrm{~kg})(x \mathrm{mg}) \\
x=125 \mathrm{mg}
\end{gathered}
$$

The physician is ordering a dose of 125 mg of diphenhydramine to be administered to the pediatric patient.

If you decide to convert from kilograms to pounds instead, you will still get the same answer.


