Medical Mathematics
Handout 1.3
Introduction to Dosage Calculations

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Now that we covered the foundation of medical mathematics in the first two handouts, we can apply those concepts for determining specific dosages to be administered to a patient.

Since the focus on these handouts is on the “mathematics” of pharmacology, I will write out the medical abbreviations and notations found in prescriptions and medication orders (instead of keeping them in their abbreviated form such as “200 mg PO bid”, I will write out 200 mg by mouth twice a day).

Basic Formula

The basic formula often used in calculating drug dosages:

$$\frac{\boldsymbol{D}}{\boldsymbol{H}} \times \boldsymbol{V} = \boldsymbol{A}$$

where

D is the **desired dose** ordered by the physician
H is **on-hand dose**, the dose that is currently available and labeled on the container
V is the **vehicle**, the medication form and amount in which the drug is supplied (e.g. tablet, pill, cream, liquid suspension, solution, aerosol metered-dose inhaler)
A is the **amount** to be given to the patient

(We will revisit these definitions when we look at the ratio and proportion method and the fractional equation method in the next two sections)

Let’s do an example together.

Example 1.3a

The physician gives an order for 1 gram of azithromycin to be administered to the patient. The hospital pharmacy has the medication available in 250 mg tablets. What is the amount needed to be administered to the patient?

Step 1: Identify the known information.

The first thing that catches our attention is that the physician's order is for 1 gram of the drug. The drugs that are available on hand are 250 mg tablets. Since the units are different, we have to first convert 1 gram to milligrams.

From our conversion, we learned that 1 gram = 1000 milligrams.

The on-hand dose (*H*) is 250 mg.
The vehicle (*V*) is 1 tablet (250 mg strength).
The desired dose (*D*) is 1 gram (1000 mg).

Now that we have all of our information, we can find our unknown (*A*).

Step 2: Plug in the known information into the formula.

$$\frac{\boldsymbol{D}}{\boldsymbol{H}} \times \boldsymbol{V} = \boldsymbol{A}$$

$$\frac{1000\ mg}{250\ mg} \times 1 = A$$

A **= 4 tablets**

4 tablets with a strength of 250 mg should be administered to the patient.

Ratio and Proportion

We discussed ratios and proportions in Handout 1.1 and began expanding on its applications when we performed conversions in Handout 1.2.

In this section we will examine the elements from the basic formula and frame them through the ratio and proportion method.

The formula which uses this method is

$$H : V :: D : x$$

If you recall from when we defined proportions, V and D are the means (the inside terms) and H and x are the extremes (the outside terms).

In this formula, H and V are the **known quantities**. D is the desired dose and x is the unknown (the amount to give).

Let's do an example together.

Example 1.3b

The physician gives an order for 600 mg gabapentin to be administered to the patient. If 300 mg capsules are available, how many capsules should be administered?

Step 1: Identify the known information.

The on-hand dose H is 300 mg.
The vehicle is 1 capsule (300 mg strength).
The desired dose is 600 mg.

Step 2: Plug in the known information into the formula.

$$300 : 1 :: 600 : x$$

If you can recall from our earlier discussions in previous handouts, proportions show the relationship between two equal ratios. This can be written as

$$\frac{300\ mg}{1\ capsule} = \frac{600\ mg}{x\ capsules}$$

We then cross-multiply:

$$(300)(x) = (600)(1)$$

$$300x = 600$$

$$\frac{\cancel{300}x}{\cancel{300}} = \frac{600}{300}$$

$$x = 2$$

2 capsules with a strength of 300 mg should be administered to the patient.

The formulas are flexible so we can also use the ratio and proportion method for our previous example, *Example 1.3a*:

$$\boldsymbol{H : V :: D : x}$$

$$250 \text{ mg} : 1 \text{ tablet} :: 1000 \text{ mg} : x$$

$$\frac{250\ mg}{1\ tablet} = \frac{1000\ mg}{x}$$

$$250x = 1000$$

$$\frac{\cancel{250}x}{\cancel{250}} = \frac{1000}{250}$$

$$x = 4$$

We get the same answer as the one we calculated in Example 1.3a. 4 tablets are to be administered.

<u>Fractional Equation</u>

For the fractional equation method, we just set up the formula from the ratio and proportion method as fractions:

$$\boldsymbol{\frac{H}{V} = \frac{D}{x}}$$

If you recall, we can solve for x through cross-multiplication. When you isolate the x you should recognize the basic formula we discussed in 1.3a.

$$\boldsymbol{\frac{H}{V} = \frac{D}{x}}$$

$$\boldsymbol{Hx = DV}$$

$$\boldsymbol{\frac{\cancel{H}x}{\cancel{H}} = \frac{DV}{H}}$$

$$\boldsymbol{x = \frac{DV}{H} = \left(\frac{D}{H} \times V\right)}$$

$\boldsymbol{x}$ will represent A, the amount of drug we need to give.

Let’s do an example together.

Example 1.3c

A physician orders 5000 units of heparin to be administered subcutaneously. If the available dose on hand is 20000 units/ml, what is the amount that will be administered to the patient?

Step 1: Identify the known information.

The on-hand dose H is 20000 units/ml.
The vehicle V is liquid (form). The concentration of heparin is 20000 units per 1 milliliter.
The desired dose D is 5000 units.

Step 2: Plug in the known information into the formula.

$$\frac{H}{V} = \frac{D}{x}$$

$$\frac{20000\ units}{1\ ml} = \frac{5000\ units}{x}$$

$$20000x = 5000$$

$$\frac{\cancel{20000}x}{\cancel{20000}} = \frac{5000}{20000}$$

$$x = \frac{1}{4} = 0.25$$

A volume of 0.25 ml of the heparin (20000 units/ml) will be prepared to get the appropriate dosage.

Again, if you use the Basic Formula that we learned in *1.3a*, you will get the same answer. The great thing about these methods is that you can choose whichever you are most comfortable with. If you do your calculations correctly, the answers should all be similar.

Dimensional Analysis

Dimensional analysis (also known as factor-label method or unit factor method) is used when performing calculations which have different units. A starting unit is converted to an equivalent quantity for a different unit. For example, if I drink 9 cups of water per day and I want to find out how many liters it equals. I would first have to convert the 9 cups to milliliters and then take that amount and convert it from milliliters to liters. This can get very involved if we use this process; however, using the dimensional analysis method (factor-label method) can make it easier to calculate.

Another important element of dimensional analysis is the **conversion factor**. The conversion factor is the numerical relationship between two units. For example, in Handout 1.2 we looked at several numerical relationships such as

1 L = 1000 ml
1 kg = 2.2 lb
1 gr (grain) = 60 mg (approx. 64.8 mg)

If you have taken science courses such as chemistry or physics, this should be familiar to you. Although it may seem intimidating, it's quite simple so don't worry!

Let's do an example using the methods we learned in Handout 1.2 when we looked at conversions.

Example 1.3d

An athlete training for a competition drank 12 cups of water in one day. How many liters is that equivalent to?

Step 1: Identify your starting quantity and related conversion factor.

We know that our starting quantity is 12 cups.

We want to convert it to liters, but we only learned the cups equivalent to milliliters. That is 1 cup = 236.588 ml (or 240 ml if you prefer).

The conversion factor between cups and milliliters should look like this (they should be equivalent):

$$\frac{1\ c}{236.588\ ml\ (240\ ml)}$$

OR

$$\frac{236.588\ ml\ (240\ ml)}{1\ c}$$

Step 2: Set up the equation so that units cancel out.

The simplest way to do this is to draw a horizontal line and then draw a line to divide it along the way as you add the conversion factors (OR you can use parentheses and write them down in fraction form). I will use both forms in the next few examples.

The most important thing to note is that the units have to **cancel out**. In order for that to work, similar units have to be diagonal to each other. The numerator of the previous column must cancel out with the denominator of the next column.

Please do not confuse this with the ratio and proportion method where similar units were aligned with each other across the equal sign (as we had done in our previous examples)! The answer will be incorrect. We will look at this after we do the problem.

First step is the conversion factor from cups to milliliters:

12 cups	236.588 ml	
	1 cup	

As we look at this we notice that we have to cancel out the milliliters (since we can cancel out the cups already). Since our final equivalent quantity is in liters, we use the conversion factor between milliliters and liters. We learned that 1 L = 1000 ml. Again, let's make sure to have the similar units (ml in our case) be diagonal to each other so that they can cancel out.

Next step is putting in the conversion factor between milliliters and liters:

12 cups	236.588 ml	1 L
	1 cup	1000 ml

As you set up your equation, the final unit that should be at the upper right (in our example it is 1 L) should match what you want your final answer to be equivalent to. In other words, our starting quantity is in cups and our final equivalent quantity will be converted to liters. As you cancel out the unwanted units, the only one that should remain should be the relevant unit we are converting to (again, we want to convert to L so the L should remain).

The final step is to perform the calculations. **When numbers are on the same side you multiply. If they are on opposite sides, you perform division (dividing top by bottom). One trick that I do is that I multiply the numbers on top first. Then, I multiply the numbers on the bottom. Afterward, I divide the top by the bottom to get the final answer. It makes the calculation easier.**

The numbers are usually simple in medical mathematics so there won't be complex manipulation of the equation. As we can see, all of the unwanted units cancel out which leaves the liter for our equivalent conversion.

Moving from left to right the numbers at the top are multiplied. (You can do the same for the bottom, but since you just need to multiply 1000 by 1, it's not necessary to do that step)

We then divide the calculated top number by the bottom number. As you perform the multiplication or division in dimensional analysis, take your time and move step by step to avoid any errors.

12 ~~cups~~	236.588 ~~ml~~	1 L
	1 ~~cup~~	1000 ~~ml~~

Part 1: Moving from left to right we **multiply**: 12 × 236.588 × 1 = 2839.056

Part 2: We then **divide** that number by 1000: 2839.056 ÷ 1000 = 2.839

Our final answer is 2.839 L.

The athlete drank around 3 L (approx. 2.839 L) of water that day.

<u>Incorrect Procedure</u>

As we discussed earlier, if the units do not cancel out due to the incorrect set up of the equation, the final answer will not be intelligible:

This is the incorrect procedure (**DO NOT FOLLOW!**):

12 cups	1 cup	1 L
	236.588 ml	1000 ml

Since 12 is the only number that is not 1 at the top, we won't need to worry about multiplying and we can focus on the bottom first.

Multiply the bottom numbers and then perform the division:

Bottom: 236.588 × 1000 = 236588

Then,

$$\frac{12}{236588} \approx 5.072 \times 10^{-5}$$

When we calculate this, we get 5.072×10^{-5} (cups)^{2}L/ml^2. This is incorrect and the units don't make sense. The amount is so small it's as if the athlete did not drink anything at all!

We can also use the ratio and proportion method as an alternative to dimensional analysis. If we had performed the conversion using our usual ratio and proportion method:

$$\frac{12\ cups}{x\ ml} = \frac{1\ cup}{236.588\ ml}$$

We cross-multiply to get

$$(12\text{ c})(236.588\text{ ml}) = (1\text{ c})(x\text{ ml})$$

$$x = 2839.056\text{ ml}$$

Now that we converted it to milliliters, we have to take this number and convert it to liters:

$$\frac{2839.056\ ml}{x\ L} = \frac{1000\ ml}{1\ L}$$

We cross-multiply to get

$$(2839.056\text{ ml})(1\text{ L}) = (1000\text{ ml})(x\text{ L})$$

$$\frac{2839.056}{1000} = \frac{\cancel{1000}x}{\cancel{1000}}$$

$$\boldsymbol{x = 2.839\text{ L}}$$

As you can see, we get the same answer similar to the dimensional analysis method. It took several steps, but other than that it is still pretty straightforward. Again, whichever method you prefer will be fine. Just note that in dimensional analysis the units have to cancel out. For ratio and proportion, the units line up with each other since we are comparing equal quantities.

To finish up this handout, let's do a few medical examples using dimensional analysis.

Example 1.3e

A 180 lb patient being treated for atrial fibrillation is to be administered digoxin. If the recommended dosage is 10 μg/kg body weight, how many milligrams of digoxin should be administered to the patient?

One thing you will notice in this example is that the dosage is in µg/kg (micrograms per kilogram). Since each person has a different weight, the dosage will be adjusted based on his or her weight.

Step 1: Identify the starting quantity and conversion factor.

The patient weighs 180 lb.
The recommended dosage is 10 µg/kg body weight.

The starting quantity will be 180 lb. We then use the conversion factor 1 kg = 2.2 lb. We also learned in Handout 1.2 that 1 mg = 1000 µg.

Step 2: Set up the equation so that the units cancel out.

180 ~~lb~~	1 ~~kg~~	10 ~~µg~~	1 mg
	2.2 ~~lb~~	1 ~~kg~~	1000 ~~µg~~

(From Column 1 to Column 2) Start with the upper left (180) and divide by 2.2: 180 ÷ 2.2 = 81.8181
(From Column 2 to Column 3) Multiply that number by 10: 81.8181 × 10 = 818.181
(From Column 3 to Column 4) Divide that number by 1000: 818.181 / 1000 = 0.8181 ≈ 0.82 mg

OR

You can multiply the numbers at the top (180 × 10 = 1800). Then multiply the numbers at the bottom (2.2 × 1000 = 2200). Finally, perform the division:

$$\frac{1800}{2200} \approx 0.81818 \approx 0.82$$

(The second method is my preferred method!)

The total dosage for a 180 lb (82 kg) patient is 0.82 mg.

We worked on multiple units in this example so dimensional analysis simplified the process. You could also do the ratio and proportion method where you will have to divide it into components. I'll quickly do it here and skip writing down the cross-multiplication steps since you are familiar with it already.

First the weight:

$$\frac{180\ lb}{x\ kg} = \frac{2.2\ lb}{1\ kg}$$

x = 81.81 kg

Next kg per µg equivalency:

$$\frac{10\ \mu g}{1\ kg} = \frac{x\ \mu g}{81.81\ kg}$$

x = 818.1 µg

Finally convert µg to mg:

$$\frac{1000\ \mu g}{1\ mg} = \frac{818.1\ \mu g}{x\ mg}$$

x = 0.8181 mg ≈ 0.82 mg

So it's not too bad if you do ratio and proportion. Dimensional analysis allows you to skip these steps and perform the calculation in one go.

For me, I prefer using this method of dimensional analysis. There is another way taught in pharmacy and nursing where you begin with the unknown (the final calculated dosage) and work backwards. The reason why I prefer the method taught in science courses such as chemistry is that it's easier to follow as you move forward.

I'll provide the other method here as reference so that you will be familiar with it also.

Dimensional Analysis (Method 2)

I'll begin with a time conversion example and then work on a medical example.

Example 1.3f

What is the equivalent value of 5000 seconds in hours?

Using this method, we start with our unknown. The unknown is the final value we want to convert to. In the example, it is hours. Let x hours equal our final value.

x hours = ?

Next, we have to find a conversion factor to help us get closer to seconds. To do this, we begin by lining up our first conversion factor so that the numerator will have the same units as our final value.

Next, we use a known conversion factor to cancel out the denominator with the numerator of the next conversion factor. We know the relationship between minutes and seconds so the next step to get closer to our answer is to find the relationship between hours and minutes.

1 hour = 60 minutes

$$(x\ hours) = \left(\frac{1\ hour}{?}\right)$$

$$(x\ hours) = \left(\frac{1\ hour}{60\ minutes}\right)$$

As you can see, the top of the first conversion factor has similar units to the final value (in our example, the units are hours).

Since we want to cancel out unwanted units and leave only the units we need for the final equivalency (hours in our example), we have to cancel out the minutes. We know that 1 minute = 60 seconds. This is our next conversion factor.

$$(x\ hours) = \left(\frac{1\ hour}{60\ minutes}\right)\left(\frac{1\ minute}{60\ seconds}\right)$$

Finally, we put our starting quantity in the numerator to cancel out the units in the previous denominator (seconds). All of the unwanted units should cancel out and we should be left with the relevant unit and our final answer.

$$(x\ hours) = \left(\frac{1\ hour}{60\ minutes}\right)\left(\frac{1\ minute}{60\ seconds}\right)(5000\ seconds)$$

$$(x\ hours) = \left(\frac{1\ hour}{60\ \cancel{minutes}}\right)\left(\frac{1\ \cancel{minute}}{60\ \cancel{seconds}}\right)(5000\ \cancel{seconds})$$

We then perform the calculations starting with our initial value. You can start by doing the following:

5000 / 60 = 83.33

83.33/ 60 = 1.3888 ≈ 1.39 hours

OR

You can multiply the numbers in the denominator 60 × 60 to get 3600. Then you can take the value at the numerator and divide (Again, this is my preferred method since you're less prone to making calculation mistakes):

$$\frac{5000}{3600} \approx 1.3888$$

5000 seconds is equivalent to 1.39 hours.

Let's move on to a medical example.

Example 1.3g

The physician orders 0.01 g of oxycodone hydrochloride solution to be administered orally to a patient suffering from moderate pain. The drug is available as a 5 mg/5 ml solution. How many milliliters should be given to the patient?

Step 1: Identify the known information.

The physician's order is for 0.01 g of the drug to be administered to the patient.
The on-hand dose of the drug that is available has a strength of 5 mg/5 ml.

Step 2: Identify the unit of measure that is being calculated.

In our example, we are trying to find milliliters. After identifying the unit of measure being calculated, we have to find a conversion factor related to it (in our example, we have to find a conversion factor related to ml).

$$ml = \left(\frac{x\ ml}{?}\right)$$

The conversion factor related to ml is the on-hand dose 5 mg/5 ml.

$$ml = \left(\frac{5\ ml}{5\ mg}\right)$$

Next we have to cancel out the units until we can find an equivalency for the desired dose. The units in the denominator of the current conversion factor must cancel out with the numerator of the next conversion factor. Our next focus is on milligrams.

$$ml = \left(\frac{5\ ml}{5\ mg}\right)\left(\frac{?\ mg}{?}\right)$$

Our desired dose is in grams. Since we are looking to cancel out milligrams, we can use the conversion factor expressing the relationship between milligrams and grams. We learned in Handout 1.2 that 1 g = 1000 mg.

$$ml = \left(\frac{5\ ml}{5\ mg}\right)\left(\frac{1000\ mg}{1\ g}\right)$$

Now that we have the conversion factor for grams, we can cancel out the units from our desired dose.

$$ml = \left(\frac{5\ ml}{5\ mg}\right)\left(\frac{1000\ mg}{1\ g}\right)(0.01\ g)$$

$$ml = \left(\frac{5\ ml}{5\ \cancel{mg}}\right)\left(\frac{1000\ \cancel{mg}}{1\ \cancel{g}}\right)(0.01\ \cancel{g})$$

Moving from left to right:
(Column 1) Divide 5 by 5 to get 1
(Column 1 to Column 2) Multiply 1 by 1000 to get 1000.
(Column 2 to Column 3) Multiply 1000 by 0.01 to get 10.

OR

Multiply the values at the top (5 × 1000 × 0.01 = 50)
Multiply the values at the bottom (5 × 1 = 5)
Divide the top by the bottom (50/5 = 10)

10 milliliters must be administered to the patient.

You can also use my preferred method that I had used in Examples 1.3d and 1.3e.

$$(0.01\ g)\left(\frac{1000\ mg}{1\ g}\right)\left(\frac{5\ ml}{5\ mg}\right) = x\ ml$$

Cancel out unwanted units and leave the ones needed for the desired dose:

$$(0.01\ \cancel{g})\left(\frac{1000\ \cancel{mg}}{1\ \cancel{g}}\right)\left(\frac{5\ ml}{5\ \cancel{mg}}\right) = x\ ml$$

Perform the calculations:

Multiply the values at the top (5 × 1000 × 0.01 = 50)
Multiply the values at the bottom (1 × 5 = 5)
Divide the top by the bottom (50/5 = 10)

Our answer is similar to our previous one above: **10 ml**

As I conclude, you can see that there are several ways to perform the calculations. Choose whichever method you are most comfortable with. As long as you follow the correct steps, you should arrive at the right conclusion.

The information that we covered in Handouts 1.1, 1.2, and 1.3 form the foundation that we will use for the following topics (e.g. dosages calculations covering oral, parenteral, intravenous, etc.).