

Medical Mathematics
Handout 1.4
Concentrations and Oral Dosage Calculations

by Kevin M. Chevalier



## Introduction

Now that we have reviewed many of the general techniques used to perform dosage calculations, we can move on to its applications.

On prescriptions and medication orders, $\mathbf{P O}$ (Latin for per os meaning "by mouth") is the abbreviation used to designate medications which are to be administered by mouth (orally).

So far the solid dosage forms we looked at included tablets and capsules. A few of the examples we worked out in the previous handout included aspirin and azithromycin tablets. In this handout we will also take a look at liquid dosage forms which are administered orally (e.g. cough syrup).

We will first examine concentrations. From the examples in our previous handouts, we looked at concentrations such as $5 \mathrm{mg} / 5 \mathrm{ml}$. We will now take a closer look at the meaning of each component.


## Concentrations

Concentration refers to the amount of solute (active ingredient) per unit of volume of solvent in the solution.

The solute is the component contained in smaller amounts in the solution. The solvent is the component contained in larger amounts in the solution. These two elements form a mixture called a solution. (The solute is dissolved in the solvent)

For example normal saline ( $0.9 \%$ ) is 9 grams of NaCl (sodium chloride) per 1 liter of solvent. We will discuss this type of concentration when we get to the $\%$ weight/volume section (solid dissolved in a liquid).

We are going to examine three types of concentrations which we will encounter more frequently in the topics following this handout. These will be weight/weight, weight/volume, and volume/volume percent concentrations.

## Weight/Weight (\% w/w)

Weight/weight (weight per weight) percent concentration measures the concentration of two substances: the numerator is the mass (weight) of the solute and the denominator is the mass (weight) of the solution.

To find the weight/weight percent concentration we use the formula:

$$
\% w / w=\frac{\text { weight of solute }(\text { in } \text { grams })}{\text { weight of solution }(\text { in grams) }} \times 100
$$

For this calculation, the focus is on the weight of the two components (the solute and the solution). Medications with \%w/w concentrations are usually semisolid substances such as creams, gels, and ointments or solid substances.

In our next example, we will look at the percentage strength of a medication. First, let's define what percentage strength means.

Percentage strength (\%), which is highlighted in the following example (Example 1.4 a ), refers to the amount of the active ingredient (in grams) per 100 grams (or ml) of the solvent.

For a medication specified in this manner (e.g. 2\%, o.5\%, etc.):

$$
\text { Percentage strength }(\%)=\frac{x g}{100 g(\text { or } 100 \mathrm{ml})}
$$

If a medication has a percentage strength of $5 \%$,

$$
\frac{5 g}{100 g}
$$

there are 5 grams of the active ingredient (the ingredient that produces an effect) per 100 grams of solvent (e.g. the cream or gel).

## Example $1.4 a$

A physician prescribes a 30 gram tube of $0.05 \%$ fluocinonide cream for a patient suffering from eczema. The physician's directions state that 15 g of the cream should be administered for one week.
a. What is the total amount of fluocinonide (the active ingredient) in the tube?
b. What is the amount of fluocinonide in the dosage which should be administered to the patient for that week?

Part A: Use proportions to find the total amount of fluocinonide in the $30 \mathbf{g}$ tube.

$$
\begin{gathered}
\frac{0.05 g(\text { active })}{100 g}=\frac{x g}{30 g(\text { total amount in tube })} \\
(x \mathrm{~g})(100 \mathrm{~g})=(0.05 \mathrm{~g})(30 \mathrm{~g}) \\
100 x=1.5 \\
\frac{100 x}{100}=\frac{1.5}{100} \\
x=0.015 \mathrm{~g}
\end{gathered}
$$

The total amount of fluocinonide in the tube is 0.015 g ( 15 mg ).

Part B: Use the percentage strength to find the required dose.
Step 1: Apply the definition of percentage strength to the medication.
For Part B, we have to use the definition of percentage strength:

$$
0.05 \%=\frac{0.05 \mathrm{~g}}{100 \mathrm{~g}}
$$

Step 2: Use proportions to find the dosage amount required for the week.
There are two ways that we could do this: (1) through the original percentage strength or (2) with the calculation from Step 1 (the amount of active ingredient in the total container).

Let's begin by using the calculation from Step 1:

$$
\begin{gathered}
\frac{0.015 g}{30 g}=\frac{x g}{15 g} \\
(30 \mathrm{~g})(x \mathrm{~g})=(15 \mathrm{~g})(0.015 \mathrm{~g}) \\
30 x=0.225 \\
\frac{30 x}{30}=\frac{0.225}{30} \\
\boldsymbol{x}=\mathbf{o . 0 0 7 5} \text { grams } \\
\text { Using the original percentage strength: }
\end{gathered}
$$

$$
\begin{gathered}
\frac{0.05 g}{100 g}=\frac{x g}{15 g} \\
(100 \mathrm{~g})(x \mathrm{~g})=(15 \mathrm{~g})(0.05 \mathrm{~g}) \\
100 x=0.75 \\
\frac{100 x}{100}=\frac{0.75}{100} \\
\boldsymbol{x}=\mathbf{0 . 0 0 7 5} \text { grams }
\end{gathered}
$$

We get the same results either way. We can see that the result for Part B can be verified through the result in Part A.

From Part A, we know that the entire tube contains 30 grams of the entire medication. In those 30 grams, there is a total of 0.015 grams of the active ingredient fluocinonide. Since the physician ordered 15 grams (half of the cream) for the entire week, we can just take half of the answer from Part A:

$$
0.015 \text { grams } \div 2=0.0075 \mathrm{~g} \text { fluocinonide }
$$

The total amount of fluocinonide in the dosage is 0.0075 g (or 7.5 mg ).

## Weight/Volume (\% w/v)

Weight/volume (weight per volume) percent concentration measures the concentration in which a solid solute is dissolved in a liquid.

$$
\% w / v=\frac{\text { weight of solute }(\text { in } \text { grams })}{\text { volume of solution }(\text { in milliliters })} \times 100
$$

We discussed an example earlier when we looked at normal saline. Normal saline used for infusions consists of 9 grams of sodium chloride ( NaCl ) dissolved in 1 L ( 1000 ml ) of distilled water.

Using the above formula:

$$
\begin{gathered}
\frac{9 \text { grams } \mathrm{NaCl}}{1000 \mathrm{~mL} \text { distilled water }} \times 100=0.9 \% \\
0.9 \% \mathrm{w} / \mathrm{v} \text { normal saline solution }
\end{gathered}
$$

For many other solutions delivered intravenously (e.g. dextrose in water), they are calculated as follows:

$$
\% w / v=\frac{\text { weight of solute (in grams) }}{100 \mathrm{ml} \text { of solution }}
$$

Now let's do an example.

## Example $1.4 b$

A hospital pharmacy is preparing a $25 \% \mathrm{w} / \mathrm{v}$ sucrose solution.
On hand the pharmacy has 500 grams of sucrose. How many milliliters of $25 \% \mathrm{w} / \mathrm{v}$ sucrose solution can be created from this?

Step 1: Gather the information from the percent concentration of the solution.

$$
25 \% w / v=\frac{25 \mathrm{~g} \text { sucrose }}{100 \mathrm{ml} \text { solution }}
$$

Step 2: Use proportions to evaluate the correct amount (in milliliters) that can be prepared.

$$
\begin{gathered}
\frac{25 \mathrm{~g}}{100 \mathrm{ml}}=\frac{500 \mathrm{~g}}{x \mathrm{ml}} \\
(25 \mathrm{~g})(x \mathrm{ml})=(500 \mathrm{~g})(100 \mathrm{ml}) \\
25 x=50000
\end{gathered}
$$

$$
\begin{aligned}
& \frac{25 x}{25}=\frac{50000}{25} \\
& x=2000 \mathrm{ml}
\end{aligned}
$$

2000 milliliters of $\mathbf{2 5 \%}$ sucrose solution can be made from 500 grams of sucrose.

## Volume/Volume (\% v/v)

Volume/volume (volume per volume) percent concentration measures the concentration in which both substances are liquids (a liquid solute is dissolved in a liquid solution).

A common example is when we have a concentrated substance and we want to dilute it (e.g. diluting 91\% isopropyl alcohol with water).

$$
\% v / v=\frac{\text { volume of solute }(\text { in milliliters })}{\text { volume of solution }(\text { in milliliters })} \times 100
$$

## Example 1.4c

The concentration of a cleaning solution is $60 \% \mathrm{v} / \mathrm{v}$. The laboratory technician needs to create 300 ml of a solution with a strength of $40 \%$. How many milliliters will be required to achieve this?

Step 1: Identify the amount of active ingredient in the diluted solution.

$$
40 \%=\frac{40 \mathrm{ml}}{100 \mathrm{ml}}
$$

$40 \% \mathrm{v} / \mathrm{v}$ of the cleaning solution will tell us that there are 40 ml of the active ingredient per 100 ml of the solution.

We first need to find how much of the active ingredient is in 300 ml of solution with a 40\% strength.

$$
\begin{aligned}
\frac{40 \mathrm{ml}}{100 \mathrm{ml}} & =\frac{x \mathrm{ml}}{300 \mathrm{ml}} \\
(100 \mathrm{ml})(x \mathrm{ml}) & =(40 \mathrm{ml})(300 \mathrm{ml})
\end{aligned}
$$

$$
\begin{gathered}
100 x=12000 \\
\frac{100 x}{100}=\frac{12000}{100} \\
x=120 \mathrm{ml}
\end{gathered}
$$

There are 120 ml of the active ingredient in 300 ml solution with a $40 \%$ strength.

Step 2: Identify the amount of the original concentrated solution needed to get the final product.

$$
\begin{gathered}
\frac{60 \mathrm{ml}}{100 \mathrm{ml}}=\frac{120 \mathrm{ml}}{x \mathrm{ml}} \\
(60 \mathrm{ml})(x \mathrm{ml})=(120 \mathrm{ml})(100 \mathrm{ml}) \\
60 x=12000 \\
\frac{60 x}{60}=\frac{12000}{60} \\
x=200 \mathrm{ml}
\end{gathered}
$$

200 ml of the original cleaning solution will be needed to create a 300 ml solution (40\% strength).

## Total Amount of Active Drug in a Package/Container

When we looked at the solid forms of drugs (e.g. tablets or capsules), each tablet or capsule specified the strength of the drug (e.g. 300 mg per tablet).

If the drug is in liquid form (measured by volume) we can find the total amount of active drug when we are given the concentration of the drug.

## Example $1.4 d$

For example if we have 150 ml bottle of amoxicillin $250 \mathrm{mg} / 5 \mathrm{ml}$, we can calculate the total amount of active drug from this information.

## Step 1: Identify the concentration of the drug.

The concentration is $250 \mathrm{mg} / 5 \mathrm{ml}$.

Step 2: Identify the size of the package or container.
The size of the container will tell you the total volume of the drug. The size of the bottle is 150 ml (the total volume).

Step 3: Find the total amount of active drug in the container.
We can find this by using the ratio and proportion method.

$$
\frac{250 \mathrm{mg}}{5 \mathrm{ml}}=\frac{x \mathrm{mg}}{150 \mathrm{ml}}
$$

$$
(5 \mathrm{ml})(x \mathrm{mg})=(250 \mathrm{mg})(150 \mathrm{ml})
$$

$$
5 x=37500
$$

$$
\frac{5 x}{5}=\frac{37500}{5}
$$

$$
x=7500 \mathrm{mg}
$$

The total amount of drug in the container is 7500 mg .

In some cases, adjustments to a drug need to be performed so that it can be taken more easily by a patient. We will go back to this example when we have to calculate a liquid formulation of a drug for a patient having difficulty with swallowing. In this case, if there is no liquid formulation of a drug available, it will need to be compounded in the pharmacy.

## Oral Dosages

Oral dosage calculations are pretty straightforward and many of the formulas and techniques which we used in the previous handout can be applied to these problems.

## Example 1.4e

A physician is treating a patient with strep throat and gives an order for Cephalexin 500 mg PO q12h $\times 7$ days (by mouth every 12 hours for one week). The hospital pharmacy has Cephalexin available in 250 mg capsules.
a. What is the daily dose that needs to be administered to the patient? b. What is the total amount of medication required to fulfill this order for the entire week?

## Step 1: Find the amount of drug needed for one dose.

There are quite a few numbers to consider for this example. However, if we move step by step, it should make it easier to work out. We first want to look at the drug strength ordered by the physician and what strength is available.

Drug strength ordered: 500 mg
Drug strength available: 250 mg per 1 tablet
You can calculate this by head and know that you will need 2 tablets to equal 500 mg , but you can also use the methods that we learned in the previous handouts.


$$
\begin{gathered}
\frac{250 \mathrm{mg}}{1 \text { tablet }}=\frac{500 \mathrm{mg}}{x \text { tablets }} \\
(250)(x)=(500)(1) \\
\frac{250 x}{250}=\frac{500}{250} \\
x=2 \text { tablets }
\end{gathered}
$$

## Step 2: Find the amount of drug needed for one day.

We know that there are 24 hours in one day. The physician orders 500 mg of Cephalexin to be administered every 12 hours.

For one day, 500 mg will be administered twice. From Step 1 we calculated that 2 capsules will be taken for one dose.
a. Since two doses are needed for one day, the patient will take 4 capsules. The total amount of medication is 1000 milligrams for one day.

Step 3: Find the total amount of drug required for the order.
Since we found the amount that will be administered to the patient for one day, we just need to multiply the amount by 7 to get the total amount of drug for the entire order.

From Step 2, we found that 1000 milligrams will be administered daily.

$$
1000 \times 7=7000 \mathrm{mg}
$$

b. 7000 milligrams of Cephalexin are needed to fulfill the physician's order.

Another important point to consider is that some patients may not be able to take a medication orally. Examples will include pediatric patients or patients who have difficulty swallowing tablets or capsules.

Earlier on we examined how to find the total amount of a drug in a container when we are given the concentration and the total volume of the medication. When a liquid formulation is not commercially available, the pharmacist can compound a new formulation from the solid form of the medication.

## Example $1.4 f$

A pediatric patient is prescribed a 350 mg tablet by a physician. The patient tells the physician that he has trouble swallowing tablets and asks if there is a liquid formulation for the drug. Since there is none available, the compounding pharmacist speaks to the physician and agrees to create a liquid formulation with a concentration of $250 \mathrm{ml} / 5 \mathrm{ml}$ and a total volume of 350 ml .

How many 300 mg tablets will be used to create a liquid formulation of the medication?

Step 1: Calculate the total amount of active drug required for the liquid formulation.

We performed these calculations earlier in Example 1.4b. We will simply use the ratio and proportion method.

$$
\begin{gathered}
\frac{250 \mathrm{mg}}{5 \mathrm{ml}}=\frac{x \mathrm{mg}}{350 \mathrm{ml}} \\
(250 \mathrm{mg})(350 \mathrm{ml})=(5 \mathrm{ml})(x \mathrm{mg}) \\
5 x=87500 \\
\frac{5 x}{5}=\frac{87500}{5} \\
x=17500 \mathrm{mg}
\end{gathered}
$$

This answer is the total amount of active drug for our liquid formulation.

Step 2: Using the amount of active drug required for the formulation (Step 1's answer), find how many tablets will be used when compounding the formulation.

$$
\begin{aligned}
\frac{350 \mathrm{mg}}{1 \text { tablet }} & =\frac{17500 \mathrm{mg}}{x \text { tablets }} \\
(350 \mathrm{mg})(x \text { tablets }) & =(17500 \mathrm{mg})(1 \text { tablet }) \\
350 x & =17500 \\
\frac{350 x}{350} & =\frac{17500}{350} \\
x & =50
\end{aligned}
$$

50 tablets ( 350 mg strength) will be used when compounding the liquid suspension formulation ( $250 \mathrm{mg} / 5 \mathrm{ml}$ ) with a total volume of 350 ml .

## Pediatric and Weight Based Dosage Calculations

In some cases, the dosage for an individual (e.g. a child or someone who may have a special condition) will be adjusted based on factors such as weight, height, age, and their current physical condition. Since children respond differently to medications compared to adults, these factors are taken into consideration so that they may receive the appropriate dose.

Let's begin with a weight based example and then find the minimum and maximum values of a dosage range for a pediatric patient.

## Example 1.49

A 9 year old pediatric patient weighing 62 pounds is being treated for acute asthma. The physician gives an order for prednisone $1 \mathrm{mg} / \mathrm{kg} /$ day PO for the next 4 days.
a. How many milligrams of prednisone will be administered daily?
b. What is the total amount (in milligrams) which will be administered to the patient for the entire 4 days?
c. If the recommended dosage is $1-2 \mathrm{mg} / \mathrm{kg} /$ day for a child under 12 years of age, find the pediatric dosage range of prednisone for a 50 lb . child.

## Part A

## Step 1: Gather the relevant information.

Dosage by body weight for prednisone: $1 \mathrm{mg} / \mathrm{kg} /$ day
Child's weight: $62 \mathrm{lb} . \approx 28.182 \mathrm{~kg}$
Conversion to recall: $1 \mathrm{~kg}=2.2 \mathrm{lb}$.

## Step 2: Set up a proportion to calculate the dose.

Based on the information from Step 1, we can set up a proportion to solve for the correct amount (in milligrams) to be administered daily. Since we learned the weight conversions between pounds and kilograms, you should be able to find the correct amount whichever way you choose.

## Using kilograms:

$$
\frac{1 \mathrm{mg}}{1 \mathrm{~kg}}=\frac{x \mathrm{mg}}{28.182 \mathrm{~kg}}
$$

$$
(1 \mathrm{mg})(28.182 \mathrm{~kg})=(x \mathrm{mg})(1 \mathrm{~kg})
$$

$$
x=28.182 \mathrm{mg}
$$

Using pounds:

$$
\begin{gathered}
\frac{1 \mathrm{mg}}{2.2 \mathrm{lb} .}=\frac{x \mathrm{mg}}{62 \mathrm{lb}} \\
(1 \mathrm{mg})(62 \mathrm{lb} .)=(x \mathrm{mg})(2.2 \mathrm{lb} .) \\
2.2 x=62 \\
\frac{2.2 x}{2.2}=\frac{62}{2.2} \\
x=28.182 \mathrm{mg}
\end{gathered}
$$

As you can see whichever way you choose (whether you use kilograms or pounds) you should receive the same answer. The important thing to note is that the units should match.

## 28.2 mg of prednisone will be administered daily for a pediatric patient

 weighing 62 lb.
## Part B

We calculated the daily dose for the pediatric patient in Part A. After performing our calculations, we found that 28.2 mg of prednisone will be administered daily. To find the total amount which will be administered for the entire 4 days, simply multiply the answer in Part A by 4.

$$
28.2 \mathrm{mg} \times 4 \text { days }=112.8 \mathrm{mg}
$$

The total amount of prednisone which will be administered to the patient for the entire 4 days is 112.8 mg .

## Part C

We are given the range for the minimum and maximum daily dose of prednisone for pediatric patients under 12 years of age.

## Step 1: Identify the minimum effective dose and maximum dose.

The minimum effective dose is the smaller value. In our example, it is $1 \mathrm{mg} / \mathrm{kg} /$ day.
The maximum dose is the larger value. In our example, it is $2 \mathrm{mg} / \mathrm{kg} /$ day.

## Step 2: Calculate the dosage range based on the given information.

Use the information from Step 1 as well as the child's weight to calculate the dosage range. The child weighs 50 lb . which is around 22.7 kg .

Let's begin with the minimum effective dose.

## Minimum Effective Dose

$$
\begin{gathered}
\frac{1 \mathrm{mg}}{1 \mathrm{~kg}}=\frac{x \mathrm{mg}}{22.7 \mathrm{~kg}} \\
(1 \mathrm{mg})(22.7 \mathrm{~kg})=(x \mathrm{mg})(1 \mathrm{~kg}) \\
x=22.7 \mathrm{mg}
\end{gathered}
$$

Next, we calculate the maximum dose.

$$
\begin{gathered}
\text { Maximum Dose } \\
\frac{2 \mathrm{mg}}{1 \mathrm{~kg}}=\frac{x \mathrm{mg}}{22.7 \mathrm{~kg}} \\
(2 \mathrm{mg})(22.7 \mathrm{~kg})=(x \mathrm{mg})(1 \mathrm{~kg}) \\
x=45.4 \mathrm{mg}
\end{gathered}
$$

## Safe Recommended Range: 22.7-45.4 mg daily

The recommended daily dosage range of prednisone for a $50 \mathbf{l b} .(22.7 \mathbf{~ k g})$ child is 22.7-45.4 mg per day.

For a 50 lb . child, the physician will give an order to administer prednisone within that range based on the office visit, examination, and diagnosis.

## Density

Density is the measure of a substance's mass per unit of volume. It is expressed as $\mathbf{g} / \mathbf{c m}^{\mathbf{3}}$ or $\mathbf{g} / \mathbf{m l}$. $\mathrm{cm}^{3}$ stands for cubic centimeters. In regions where the metric system isn't as widely used, cc is used for cubic centimeters.

The formula for density ( $\rho$, the lowercase Greek letter for Rho) is as follows:

$$
\rho=\frac{\text { mass }}{\text { volume }}
$$

The standard used for measurements is the density of water: $1.00 \mathrm{~g} / \mathrm{ml}$ [or $1.00 \mathrm{~g} / \mathrm{cm}^{3}$ ] of $\mathrm{H}_{2} \mathrm{O}$ at $4^{\circ} \mathrm{C}$.

If we are measuring a liquid in a lab with a volume of 35 ml and has a weight of 50 g , we can find the density as follows:

$$
\begin{gathered}
\rho=\frac{50 \mathrm{~g}}{35 \mathrm{ml}} \\
=1.43 \mathrm{~g} / \mathrm{ml} \text { or } 1.43 \mathrm{~g} / \mathrm{cm}^{3}
\end{gathered}
$$

The liquid has a volume of $1.43 \mathrm{~g} / \mathrm{ml}$ (or $1.43 \mathrm{~g} / \mathrm{cm}^{3}$ ).
Recall from Handout 1.2: $\mathbf{1} \mathbf{~ m l}=\mathbf{1} \mathbf{c c}\left(\mathrm{cm}^{3}\right.$ in the metric system).

## Specific Gravity

Now that we looked at density, we can move on to specific gravity. Specific gravity refers to the ratio of a specific substance's density to the density of a standard of reference.

The standard of reference we will use comes from the previous section on density. The mass of 1 ml of water at $4^{\circ} \mathrm{C}$ is 1 g .

The formula for specific gravity is noted as

$$
\text { Specific Gravity }=\frac{\text { density of substance/object }}{\text { density of equal volume of water }}
$$

If the specific gravity is less than 1 , the substance will float in water (substance is lighter than water). If the specific gravity is greater than 1 , it will sink in water (substance is heavier than water).

The specific gravity of a substance is constant at a specified temperature regardless of the mass or volume. I'll explain this more after this example.

## Example 1.4h

A hospital utilizes ethanol to disinfect surfaces after use. Consider the following information: at $25^{\circ} \mathrm{C}$, ethanol has a specific gravity of $0.787 \mathrm{~g} / \mathrm{cm}^{3}$. If the hospital staff has a bottle of ethanol with a volume of 575 ml , find the mass of the ethanol.

$$
\begin{gathered}
\text { Specific Gravity }=\frac{\text { density of ethanol at } 25^{\circ} \mathrm{C}}{\text { density of water }} \\
0.787=\frac{x \text { grams }}{575 \mathrm{ml}} \\
x=(0.787)(575) \\
x=452.53 \text { grams }
\end{gathered}
$$

## The mass of the ethanol in the container is $\mathbf{4 5 2 . 5 3}$ grams.

Let's go back to the point I made before the example. We learned that the specific gravity of a substance is constant at a given temperature. At $25^{\circ} \mathrm{C}$, whether we have 575 ml of ethanol or 1000 ml of ethanol, the specific gravity will always be $0.787 \mathrm{~g} / \mathrm{cm}^{3}$.

