Medical Mathematics
Handout 1.5
Ratio Strength and Dilutions

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Introduction

We will continue our applications of mathematics to the healthcare field as we progress to our next two topics: **ratio strength** and **dilutions**.

In some cases, a pharmacist has a concentrated version of a drug and needs to dilute it for a specific patient. Similarly, products such as cleansers or concentrated solutions may be diluted before use at a facility or laboratory.

Let's review a few key points before we begin.

If a medication has a strength of 0.2% w/w, it can be expressed as

$$\frac{0.2\ g}{100\ g}$$

Recall that when there is a percentage, you take the value in front of the percent and place it over one hundred. To convert the percentage to decimal form, move the decimal point two places to the left. If the decimal point is not visible (e.g. 5%, 36%, 100%), the decimal point is understood to be at the end of the number.

5% = 0.05

36% = 0.36

100% = 1.00

When we discussed ratios in earlier handouts, they were expressed in the form **(A:B)**. This describes the relationship between two numbers (in this case, A and B). This was also covered during the previous handout when we looked at concentrations and percentage strength: weight/weight, volume/volume, and weight/volume.

Concentrations describe the relationship between the active ingredient and the inactive ingredient (ratio of active to inactive ingredient). A few examples we looked at included dextrose dissolved in water to treat low blood sugar (hypoglycemia) and creams and ointments for cases such as eczema or psoriasis.

We will extend the applications of ratios and proportions to the two major topics in this handout.

Ratio Strength

Ratio strength refers to the concentration of weak or diluted solutions. Ratio strength can be converted to percentage strength as well as the opposite. Ratios must be simplified (reduced) to lowest form. In the majority of cases, the numerator will equal 1 (as we will see in the following example).

One exception is 4:5 which converts to 0.8%.

For example, if we have lidocaine gel with a percentage strength of 4%, we can convert it to ratio strength notation:

First express the percentage in ratio form:

$$\frac{4}{100}$$

Next, form a proportion by equating the percent ratio to 1 part in *x* parts:

$$\frac{4}{100} = \frac{\mathbf{1}\ \boldsymbol{part}}{\boldsymbol{x}\ \boldsymbol{parts}}$$

$$\frac{4}{100} = \frac{\mathbf{1}}{\boldsymbol{x}}$$

$$4x = 100$$

$$x = \frac{100}{4}$$

$$x = 25$$

The ratio strength of the medication is 1:25 (1 in 25)

This notation tells us that there is 1 part active ingredient in 25 parts of the solvent.

Here is a more detailed explanation when you are given the percentage strength of the medication or solution:

<u>Convert Percentage Strength to Ratio Strength</u>

Step 1: Write the percentage in fraction form. Note that the numerator is the value before the percent sign and the denominator is 100.

$$0.4\% = \frac{0.4}{100}$$

Step 2: Equate the percentage to 1/x. The simplified form of common ratio strengths has 1 in the numerator. In the example from Step 1, we will write it as the following:

$$\frac{0.4}{100} = \frac{1}{x}$$

Step 3: Cross multiply and use algebra to solve for *x*.

$$0.4x = 100$$

$$x = \frac{100}{0.4}$$

$$x = 250$$

$$0.4\% = 1{:}250$$

The ratio strength result will read as 1 part in 250 parts.

<u>Convert Ratio Strength to Percentage Strength</u>

Similarly if we are given the ratio strength of a medication or solution, we can convert it to percentage strength. We follow similar steps to the process of converting percentage strength to ratio strength. The only difference is that the unknown is the numerator of the percentage.

If a medication has a ratio strength of 1:50, we can convert it to a percentage strength. Form a proportion using the given ratio and equate it with the percent ratio with respect to 100.

$$\frac{1}{50} = \frac{\boldsymbol{x}}{\mathbf{100}}$$

$$50x = 100$$

$$x = \frac{100}{50}$$

$$= 2$$

$$x = 2\%$$

The *x* will be expressed as a percent.

<u>**Converting Ratio Strength to Concentration Strength**</u>

These conversion methods are important when we have to convert to certain medications into milligrams per milliliters and vice versa. To prevent medication errors, concentration strengths have become the standard instead of utilizing ratio strength expressions.

Let's take a look at an example of this type of conversion.

<u>*Example 1.5a*</u>

A dilution is being prepared using isoproterenol hydrochloride injection with a ratio strength of 1:5000 for intravenous administration. Updated regulations have rewritten the ratio strength as a concentration format to reduce the number of medication administration errors. How many milligrams of the drug are in 1 ml?

Step 1: Convert the ratio strength into percentage strength. Recall the definition of percentage strength:

$$\% \, {}^{\boldsymbol{w}}/_{\boldsymbol{w}} = \frac{\boldsymbol{x}\ \boldsymbol{g}}{\mathbf{100}\ \boldsymbol{g}} \ \boldsymbol{or}$$

$$\% \, {}^{\boldsymbol{w}}/_{\boldsymbol{v}} = \frac{\boldsymbol{x}\ \boldsymbol{g}}{\mathbf{100}\ \boldsymbol{ml}} \ \boldsymbol{or}$$

$$\% \, {}^{\boldsymbol{v}}/_{\boldsymbol{v}} = \frac{\boldsymbol{x}\ \boldsymbol{ml}}{\mathbf{100}\ \boldsymbol{ml}}$$

$$1:5000 = \frac{1\ g}{5000\ ml}$$

$$\frac{1\ g}{5000\ ml} = \frac{1000\ mg}{5000\ ml}$$

NOTE: when rewriting the ratio strength into percentage strength, the numerator is in grams. For the "parts", use the units that match with the percentage strength's units. If you want to convert the unit, (i.e. from grams to milligrams) you can do it after performing the calculations using the percentage strength's standard unit.

In our example we want the final result in milligrams. After using the % w/v definition, the result is in grams per milliliters. We simply convert 1 gram to milligrams.

Step 2: Set up a proportion and solve for x.

Since we know that there is 1 mg of isoproterenol in 5 ml, we can use proportions to find the amount of the drug in 1 ml.

$$\frac{1000\ mg}{5000\ ml} = \frac{1\ mg}{5\ ml}$$

$$\frac{1\ mg}{5\ ml} = \frac{x\ mg}{1\ ml}$$

$$x = \frac{1}{5}$$

= 0.2 mg

There is 0.2 mg of isoproterenol hydrochloride in 1 ml.

Let's do another example.

Example 1.5b

A dentist is using a diluted epinephrine solution with a ratio strength of 1:200 000 as an anesthetic. How many milligrams of epinephrine are contained in 1 milliliter?

The first thing we have to do is convert the ratio strength to percentage strength.

$$1:200\,000 = \frac{1\ g}{200\,000\ ml}$$

$$\frac{1\ g}{200\,000\ ml} = \frac{x\ g}{100\ ml}$$

Cross multiply and solve for *x*:

$$200\,000x = 100$$

$$x = \frac{100}{200\,000}$$

$$x = 0.0005\ g$$

Note that there is 0.0005 grams in 100 ml. We have to perform another conversion to get the final result in milligrams per milliliter.

$$\frac{0.0005\ g}{1\ g} = \frac{x\ mg}{1000\ mg}$$

$$x = 0.5\ mg$$

We then substitute the result back into the proportion:

$$\frac{0.5\ mg}{100\ ml} = \frac{x\ mg}{1\ ml}$$

$$100x = 0.5$$

$$x = \frac{0.5}{100}$$

$$x = 0.005\ mg$$

There is 0.005 mg of epinephrine per milliliter.

Parts Per Million (ppm) /Billion (ppb)

When discussing ratio strength, parts per million and parts per billion are used when the amount of active ingredient is tiny (very diluted).

$$ppm = \frac{x}{1\ 000\ 000}$$

$$ppb = \frac{x}{1\ 000\ 000\ 000}$$

A common example is fluoride in toothpaste. The standard toothpaste in the United States contains 0.15% w/v fluoride ion. Let's convert this concentration from percentage strength to parts per million.

Note the standard measurement equivalency:

$$1\ ppm = \frac{1\ mg}{1\ L}$$

Step 1: Express the percentage strength as a ratio:

0.15% w/v =

$$\frac{0.15\ g}{100\ ml}$$

Step 2: Find the amount of the active ingredient per 1 L. (Note: **1 L = 1000 ml**)

To do this, form a proportion and solve for *x*:

$$\frac{0.15\ g}{100\ ml} = \frac{x\ g}{1000\ ml}$$

$$100x = 150$$

$$x = \frac{150}{100}$$

$$= 1.5\ g$$

From this result we know that there are 1.5 grams per 1 liter.

Step 3: Convert the result from grams to milligrams.

Since parts per million is equivalent to 1 milligram per liter, we have to convert the result from Step 2 into milligrams.

$$\frac{1.5\ g}{1\ g} = \frac{x\ mg}{1000\ mg}$$

x = 1500 mg

We now have our final result.

Step 4: Apply the conversion to the final result.

Since we know that

$$\boldsymbol{x}\ \textbf{ppm} = \boldsymbol{x}\ \textbf{mg/L}$$

We plug in our values into the standard equivalency:

$$1500\ \text{ppm} = 1500\ \text{mg/L}$$

The fluoride concentration in the toothpaste is 1500 ppm.

Dilutions

We began to discuss dilutions in the final example from the previous section. When drugs or solutions are concentrated, diluted preparations are made to be tailored for the specifications of a procedure or a patient.

A common example is when you mix powdered juice into a glass of water. When the juice is too sweet, you continue to add water until the drink is balanced to your taste. Notice that the amount of powdered juice does not change. You are not adding more powdered juice, only water. Similarly, when solutions or drugs are diluted the amount of solute (e.g. the medication) remains the same, but the volume changes.

The change in volume reduces the strength of the solution. If you keep the amount of powdered juice constant (not adding any more), adding more water will dilute the juice. Similarly, adding more of the diluent to a cream or a drug will reduce its strength.

Let's take a look at an example.

Example 1.5c

A pharmacist has 30 grams of 2% antifungal cream and combines it with 20 grams of a cream base. Calculate the change in percentage strength.

Step 1: Find the amount of medication in the cream.

Since we know the medication's strength is 2%, we can apply the definition of percentage strength:

$$\frac{2\ g}{100\ g}$$

Next, use proportions to find the amount of medication in 20 grams of the cream:

$$\frac{2\ g}{100\ g} = \frac{x\ g}{30\ g}$$

$$100x = 60$$

$$x = \frac{60}{100}$$

$$= 0.6\ \text{g}$$

There is 0.6 grams of active ingredient in the 2% cream.

Step 2: Combine the amount of the diluent with the current medication to find the new volume.

We are adding 30 grams to the original 20 grams.

30 + 20 = 50 grams

Step 3: Calculate the new percentage strength using the amount of medication and the combined volume.

$$\frac{0.6\ g}{50\ g} = \frac{x\ \%}{100\ g}$$

$$50x = 60$$

$$x = \frac{60}{50}$$

$$x = 1.2\%$$

The cream now has a percentage strength of 1.2%.

<u>Dilutions Involving Solids</u>

In Example 1.5c, we had a concentrated cream and diluted it by adding additional cream base.

Let's take a look at an example where the amount of base added is unknown.

Example 1.5d

A pharmacist has 2% diphenhydramine topical ointment on hand. She needs to dilute it to create 500 grams of the ointment with a percentage strength of 1%.

a) What is the amount of the original ointment she will need?

b) How much ointment base does she need to add to make the final medication?

Step 1: Find the amount of active ingredient in the final amount of the medication.

We know that there is 1% diphenhydramine or 1 gram active ingredient per 100 grams of the carrier base in the diluted ointment.

$$\frac{1\ g}{100\ g} = \frac{x\ g}{500\ g}$$

$$100x = 500$$

$$x = 5\text{ g}$$

There are 5 grams of diphenhydramine in 500 grams of the 1% ointment.

Step 2: Use the value of the original concentrated medication to find the amount needed to be added to the inactive base.

$$\frac{2\ g}{100\ g} = \frac{5\ g}{x}$$

$$2x = 500\text{ g}$$

$$x = \frac{500}{2}$$

$$= 250\text{ grams}$$

She is going to need 250 grams of the original 2% diphenhydramine ointment. (answer to a)

Step 3: Subtract the value of the original medication (from Step 2) from the total value of the diluted ointment.

The final amount she wants is 500 grams of a 1% ointment.

500 g - 250 g = 250 g

She will need to add 250 grams of the ointment base to 250 grams of the 2% ointment to make 500 grams of the 1% ointment. (answer to b)

The steps from Example 1.5d are quite involved, but we will now learn a formula to facilitate the process:

$$Q_1V_1 = Q_2V_2$$

It can also be expressed as

$$Q_1C_1 = Q_2C_2$$

Q_1V_1 or Q_1C_1 is the initial product that is going to be diluted. Q_1 represents the quantity or amount of the product. V_1 or C_1 is the concentration (percentage strength) of the original medication or solution.

Q_2V_2 or Q_2C_2 is the final diluted product. Similarly, Q_2 represents the quantity or amount of the product. V_2 or C_2 is the concentration (percentage strength) of the diluted medication or solution.

Let's go back to the previous example and apply the formula.

The original medication is 2% diphenhydramine ointment. V_1 (or C_1) = 2% (.02)

The quantity of the diluted ointment the pharmacist wants is 500 grams. Q_2 = 500 g

The percentage strength of the diluted ointment is 1%. V_2 (or C_2) = 1% (.01)

With the three known quantities, we can now solve for the unknown variable, Q_1 (or C_1). Don't forget to convert the percentages to decimal form.

$$(0.02)x = (500)(0.01)$$

$$0.02x = 5$$

$$x = \frac{5}{0.02}$$

$$= 250 \text{ g}$$

Using the formula, we were able to determine the amount of the original medication that will be needed for the dilution. The formula allows us to skip all the way to Step 3! All we have to do is find the amount of inactive base that needs to be added to the amount of the original medication.

Let's do a new example using this formula.

Example 1.5e

A pharmacist has 100 grams of 2% miconazole nitrate topical cream on hand. He combines it with 30 grams of the cream base. What is the percentage strength of the diluted cream?

This example is similar to 1.5c. However, we will use the formula this time to find the percent strength.

The quantity of the original medication is 100 grams. $Q_1 = 100$

The strength original medication is 2% miconazole topical cream. V_1 (or C_1) = 2% (.02)

The quantity of the diluted ointment the pharmacist wants is 130 grams. He is adding 30 grams of cream base to the original 100 grams: 100 + 30 = 130. $Q_2 = 130$ g

The unknown variable is the percentage strength of the diluted cream. V_2 (or C_2) = x%

$$(100)(0.02) = (130)(x)$$

$$2 = 130x$$

$$x = \frac{2}{130}$$

$$= 0.015$$

Multiply the result by 100 to get the percentage strength.

$$0.015 \times 100 = 1.5\%$$

130 grams of the diluted miconazole cream has a percentage strength of 1.5%.

As we can see, the formula helps us save time. However, both methods work to get you to the final result. Choose whichever works best for you.

If you want to see a previous example (***Example 1.5c)*** using the formula:

$$(30)(0.02) = (50)(x)$$

$$0.6 = 50x$$

$$x = \frac{0.6}{50}$$

$$= 0.012$$

Multiply by 100 to get the percentage strength: $0.012 \times 100 = 1.2\%$

Compare that result to *Example 1.5c*.

<u>Liquid Dilutions</u>

Now that we covered dilutions of solid substances in depth, we can move on to liquid dilutions. If you have a measured amount of concentrated solution (e.g. bleach or alcohol) and continue to add water, the concentration is affected. In this case, more solvent (the water) is added while the solute (the bleach or alcohol) remains the same. The concentration of the solute is reduced by adding more solvent.

When we looked at the examples for solid dilutions, adding more of the cream or ointment base reduced the percentage strength of the medication.

<u>Example 1.5f</u>

750 ml of dextrose 50% is diluted to make 1000 ml. Calculate the new concentration of the solution.

The process here is similar to the calculations we performed previously. First let's use the formula method.

$$Q_1C_1 = Q_2C_2$$

The quantity of the starting product is 750 ml. Q_1 = 750 ml

The strength of the starting product is 50%. $C_1 = 0.50$

The quantity of the final diluted product is 1000 ml.

The strength of the final diluted product is unknown. $C_2 = x$

With the information we can solve for x:

$$(750)(0.50) = (1000)(x)$$

$$375 = 1000x$$

$$x = \frac{375}{1000}$$

$$= 0.375$$

$$0.375 \times 100 = 37.5\%$$

The diluted solution has a concentration of 37.5%.

Another method is to use the long process from *Example 1.5c.*

Dextrose 50% contains 25 grams in 50 ml solvent.

$$\frac{25\ g}{50\ ml} = \frac{x\ g}{1\ ml}$$

$$50x = 25$$

$$x = 0.5 \text{ g (or 500 mg)}$$

We now know that there are 0.5 grams or 500 mg of dextrose in 1 ml of solvent.

Next we have to find the amount of the solute (dextrose) in 750 ml of the solution.

$$\frac{0.5\ g}{1\ ml} = \frac{x\ g}{750\ ml}$$

$$x = 375 \text{ g (or 375 000 mg)}$$

In the problem, 250 ml of a solvent is added to the original 750 ml of the dextrose solution. This brings the amount to 1000 ml. Note: the amount of solute remains the same. Thus, there are 375 g of dextrose in 1000 ml of the diluted solution. Using this information we can calculate the percentage strength:

$$\frac{375\ g}{1000\ ml} = \frac{x\ \%}{100}$$

$$1000x = 37500$$

$$\boldsymbol{x = 37.5\%}$$

We get the same result as the one using the formula method.

Example 1.5g

A facility has 2 liters of a cleaning agent solution with a concentration of 80%. The instructions state to dilute the amount to 50% using distilled water. What will be the quantity of the final diluted solution? How much distilled water will be added to the agent to create the final solution? Express the answers in milliliters.

Let's use the formula to evaluate this problem.

$$\boldsymbol{Q_1V_1 = Q_2V_2}$$

The original quantity of the cleaning agent is 2 L. From the conversions, we know that 1 L = 1000 ml. Thus, the facility has 2000 ml of the cleaning agent on hand. Q_1 = 2000 ml

The concentration of the cleaning agent is 80%. V_1 = 0.8

The quantity of the final diluted solution is the unknown. $Q_2 = x$

The concentration of the final diluted solution is 50%. V_2 = 0.5

With the given information, we can solve for x:

$$(2000\ \text{ml})(0.8) = (x)(0.5)$$

$$1600 = 0.5x$$

$$x = \frac{1600}{0.5}$$

$$= 3200\ \text{ml}$$

The quantity of the final diluted solution is 3200 ml. Q_2 = 3200

To find the quantity of distilled water that is added to create the diluted solution, subtract the quantity of the original cleaning agent from the final diluted solution:

$$3200 \text{ ml} - 2000 \text{ ml} = \mathbf{1200 \text{ ml}}$$

1200 ml of distilled water were added to the cleaning agent to create the final diluted solution.

You could also perform the steps using liters and then perform the conversions at the end.

$$(2 \text{ L})(0.8) = (x)(0.5)$$

$$1.6 = 0.5x$$

$$x = \frac{1.6}{0.5}$$

$$= 3.2 \text{ L}$$

$$3.2 \times 1000 = 3200 \text{ ml}$$

For the quantity of distilled water added:

$$3.2 - 2 = 1.2 \text{ L}$$

$$1.2 \times 1000 = 1200 \text{ ml}$$

The great thing about mathematics is that there are usually different approaches to solve the problem. This flexibility is quite useful in case you forget how to do one method!

Example 1.5h

A laboratory has 30% w/v sodium chloride solution. What amount of the original solution is required to create 300 ml of a 100 mg/ml diluted solution?

Step 1: Find the percentage strength of the final diluted solution.

The first thing we will notice is that the concentration of the diluted solution is not in its percentage strength form. In addition, the numerator is written in milligrams. Since the numerator of percentage strength is in grams, we first have to convert 100 mg to grams.

$$100 \text{ mg} \div 1000 = 0.1 \text{ grams}$$

$$\frac{0.1\ g}{1\ ml}$$

With this result, we can find the percentage strength:

$$\frac{0.1\ g}{1\ ml} = \frac{x\%}{100\ ml}$$

$$x = 10\%$$

Step 2: Use the formula to find the quantity of original solution needed.

The quantity of the original quantity is the unknown. $Q_1 = x$

The concentration of the sodium chloride solution is 30%. $C_1 = 0.3$

The quantity of the diluted solution is 300 ml. $Q_2 = 300$ ml

The concentration of the diluted solution is 10%. $C_2 = 0.1$

$$(x)(0.3) = (300)(0.1)$$

$$0.3x = 30$$

$$x = 100 \text{ ml}$$

100 ml of the 30% sodium chloride solution is needed to create 300 ml of a 10% diluted solution.