Medical Mathematics
Handout 1.5a
Alligations

by Kevin M. Chevalier

Introduction

In Handout 1.5, we looked at the applications of dilutions in the scientific and healthcare fields. Concentrated stock solutions and drugs are diluted based on the needs of a patient or for a specific task. In these cases, the concentrated product was diluted using a base (e.g. ointment base) or a solution (e.g. normal saline).

We will extend these applications to **alligations**. With alligations, different strengths of the same ingredient are combined in order to produce a product with a new strength. Diluents such as sterile water can also be used to reduce the concentration strength. This is important when a concentration of a solution or medication has to be customized for purposes (e.g. prescription for a specific patient's needs or for use in a laboratory setting).

For example if you have two strengths of ethanol (ethyl alcohol) on hand: 70% and 25% and you need to produce 500 ml of ethanol with a strength of 50%, how should you proceed? We can apply the alligation method for this procedure.

Alligation Procedure

In order to begin the allegation process, we have to construct a diagram. First divide the diagram into three columns (vertical) and four rows (horizontal).

The left column is the percentage strength of the solution or medication.

The right column is the number of parts needed to produce the final product.

The top row involves the higher concentration.

The bottom row involves the lower concentration.

The middle column is the final percentage.

The fourth row is the total number of parts required (adding together #4 and #5).

<u>Alligation Diagram</u>

Percentage Strength		**Parts Required**
1. Higher Concentration Solution or Drug		4. Required Parts of Higher Concentration
	3. Final Product Concentration Strength	
2. Lower Concentration Solution or Drug		5. Required Parts of Lower Concentration
		6. Total Parts

!! Important Point: When diluting a substance with water, the water's percentage strength equals <u>zero</u>. !!

Also note that the value of the final product concentration strength (#3) must be a value between the higher (#1) and lower (#2) concentrations that will be combined.

If you have 95% and 60% strength solutions on hand, you cannot use this process to produce a 25% solution.

However, you will be able to create a 75%, 80%, or any percentage strength falling between 95% and 60%.

Let's take a look at the example from the introduction and apply it to the allegation diagram. We will then work on finding the final product step by step.

<u>Example 1.5a-1</u>

A lab assistant has two strengths of ethanol on hand: 70% and 25%. He is instructed to produce 500 ml of 50% ethanol. Calculate the amount of each of the two initial concentrations he will need to produce the final product.

Step 1: Plug in the known values into the alligation diagram.

Percentage Strength		Parts Required
70		4. Required Parts of Higher Concentration
	50	
25		5. Required Parts of Lower Concentration
		6. Total Parts

Step 2: Find the values of the required parts (#4 and #5 in the diagram) and their total (#6 in the diagram).

To do this we will use subtraction. Move diagonally and subtract the values. The values will all be positive. This is because when you try to find the value of #4 you will see that you get a negative value. Don't worry about that. Just reverse the sign. If you prefer you can subtract with the higher value first. You will get the same value either way.

#4 Required Parts of Higher Concentration: 25 - 50 (or 50 - 25) = **25 parts**

#5 Required Parts of Lower Concentration: 70 - 50 = **20 parts**

#6 Total Required Parts (#4 + #5 = #6): 25 + 20 = **45 parts**

Percentage Strength		Parts Required
70		25
	50	
25		20
		45

Step 3: Find the amount of the two strengths on hand that will be required to produce the final product.

To calculate the amounts of each strength needed, form a ratio and multiply by the volume of the total product:

$$\frac{Higher\ Concentration\ Part\ Required}{Total\ Parts} \times (Final\ Product\ Volume)$$

$$\frac{Lower\ Concentration\ Part\ Required}{Total\ Parts} \times (Final\ Product\ Volume)$$

For the amount of higher concentration needed:

$$\frac{25}{45} \times (500\ ml) \approx 277.7\ \approx 278\ ml$$

For the amount of lower concentration needed:

$$\frac{20}{45} \times (500\ ml) \approx 222.2\ \approx 222\ ml$$

Thus we have our final answer.

278 ml of 70% ethanol should be combined with 222 ml of 25% ethanol to produce 500 ml of 50% ethanol.

<u>*Example 1.5a-2*</u>

Let's take a look at an example involving dilution with water.

A pharmaceutical researcher has 50% dextrose on hand and sterile water. She wants to create 800 ml of 30% dextrose. Calculate the amount of each of the initial concentrations she will need to produce the final product.

The process will be similar to what he had done for the previous example. The only difference is that we will combine it with water instead of a different strength of the solution/drug.

Step 1: Plug in the known values into the alligation diagram.

Since we are working with sterile water, the percentage strength for the lower concentration is 0.

Percentage Strength		Parts Required
50		4. Required Parts of Higher Concentration
	30	
0		5. Required Parts of Lower Concentration
		6. Total Parts

Step 2: Find the values of the required parts (#4 and #5 in the diagram) and their total (#6 in the diagram).

#4 Required Parts of Higher Concentration: 0 - 30 (or 30 - 0) = **30 parts**

#5 Required Parts of Lower Concentration: 50 - 30 = **20 parts**

#6 Total Required Parts (#4 + #5 = #6): 30 + 20 = **50 parts**

Percentage Strength		Parts Required
50		30
	30	
0		20
		50

Step 3: Find the amount of the two strengths on hand that will be required to produce the final product.

For the amount of higher concentration needed:

$$\frac{30}{50} \times (800\ ml) = 480\ ml$$

For the amount of lower concentration needed:

$$\frac{20}{50} \times (800\ ml) = 320\ ml$$

480 ml of 50% dextrose should be combined with 320 ml of sterile water to produce 800 ml of 30% dextrose.

Alligations (Three or More Strengths of the Active Ingredient)

When performing alligations involving three or more of the same active ingredient, we can calculate the percentage strength as follows:

1. Divide the values into two columns. The left column will be the volume of the solution or medication. The right column will be the amount of active ingredient in the solution or medication.

Recall that the right column is calculated by multiplying the volume (Left Column) by the percentage strength. Do not forget to convert the percentage strength to decimal form.

For example, on hand I have 50 ml of 85% Solution A, 200 ml of 50% Solution B and 350 ml of 10% Solution C.

For the right column calculations:

(50 ml)(0.85) = 42.5 g

(200)(0.5) = 100 g

(350)(0.1) = 35 g

Amount (Volume)	Active Ingredient
50 ml	**42.5 g**
200 ml	**100 g**
350 ml	**35 g**
600 ml	**177.5 g**

2. Add the values of the left column. Then, add the values of the right column.

Total Volume: 50 + 200 + 350 = 600 ml

Total Amount of Active Ingredient: 42.5 + 100 + 35 = 177.5 g

3. Use the proportion method to convert the ratio into percentage strength. (*x* is expressed in percent form)

$$\frac{177.5\ g}{600\ ml} = \frac{x}{100}$$

$$(600 \text{ ml})(x) = (177.5 \text{ g})(100)$$

$$600x = 17750$$

$$x = \frac{17750}{600}$$

$$\approx \mathbf{29.58\%}$$

The percentage strength of the combined solution is around 30%.